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How Effective Is the Meaning Method?

A Report of the Los Angeles Study

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How effective is the meaning method in teaching arithmetic? The answer to this perplexing question still remains uncertain. It is apparent that despite the great amount of literature written on the benefits to be derived from the use of the meaning method by the students, little research has been initiated to justify the great emphasis upon this approach. Most theorists on methods of teaching arithmetic appear to be united in stressing the meaning method. However, are their recommendations on the instruction of arithmetic based upon experimental evidence or are they mere postulation?

The Los Angeles Study has recently been completed to investigate more throughly the effectiveness of the meaning method. Before considering the experiment, a discussion of the similarities and differences of the rule method and the meaning method is necessary.

The Rule Method versus the Meaning Method

Although learning in arithmetic can be facilitated in many ways, two predominant methods are used currently. The first, *the rule method*, is a technique in which an instructor solves a problem and describes the specific rules to be learned to obtain the solution to the problem. For instance, to show the class how to add $\begin{array}{r}67\\ \underline{24}\end{array}$, the instructor explains that the 7 and the 4 are added to get a total of 11. The 1 is put underneath the 4 and the other 1 is "carried." Then the 6 and the 2 are added to the 1 which was carried for a total of 9. The 9 is placed in front of the 1 giving the final answer of 91. A sufficient number of problems is explained to the students to cover other types of addition. A similar procedure is followed for computation of all other types of arithmetic problems. Thus the student is shown how to compute any specific problem by using the designated rule or rules necessary to obtain the correct solution.

In the second, *the meaning method*, the instructor explains a problem by reference to concrete examples, making use of definitions and principles of arithmetic. For example, the decimal system is explained as a number system based on ten, with the digits—zero, one, two, three, four, five, six, seven, eight, and nine. The students are told the function of the place value of each number, so that a logical basis exists for the size relationships of a group of digits despite its complexity. Thus, the number, 962 is not explained as a group of meaningless and unrelated digits, but as a group of digits in which

each has its own particular significance. The 9 represents nine hundred, the 6 represents six tens and the 2 represents two units.

In the previous problem given above, the carrying of the 1 is not explained as a rule to be memorized, but as a specific example based upon the definition of number and principles of place value. When the 7 and 4 are added to obtain 11 units, the 1 in the units column remains in the final answer of the problem. The 1 in the tens column, however, is not considered as one single unit but as a 1 in the tens column representing ten units. Thus the 1 ten is added to the 6 tens and the 2 tens to obtain a total of 9 tens. The final answer is explained as 9 tens and 1 unit or 91. Additional problems are explained in a similar manner. Students are continually encouraged to make useful generalizations from the basic concepts and principles to gain insight into arithmetical operations. Thus the instructor provides the students with a sound basis of definitions and principles of arithmetic to enable them to comprehend the processes that they utilize in the computation of arithmetic problems.

The two methods differ in several respects: the former places emphasis upon the *rules* for learning arithmetic, the latter upon *meaning* and *understanding*. The rule method is concerned with the procedure for working the problem. It presents in a brief interval of time the necessary elements to compute the desired problem. The meaning method, on the other hand, offers the student an integration of the concepts and principles of arithmetic as well as the computation of the problem. Explanations of "why" the processes work are given to the student. Rules are explained, not in isolated segments, but as conclusions based upon arithmetical definitions and principles.

The rule method has its foundations in connectionism or stimulus-response psychology. The meaning method, which is a newer approach that has gained wide acceptance, is based on the principles of gestalt psychology.

In the evaluation of the meaning method made by theorists in arithmetic learning, the usual procedure has been to compare the meaning method with the rule method. Brownell and Chazel (3) were among the first to investigate the problem with an experiment that demonstrated the ineffectiveness of premature drill in the elementary level. Thiele (8) and later Swenson (7) made experiments based on learning the 100 addition facts which showed significant differences in ability to make generalizations in arithmetic problems by those students who were instructed by the meaning method. The "discovery" method was compared with the "traditional" approach in an investigation by McConnell (6) which used the addition facts as the criterion of learning. He found that the discovery method was superior for generalization but was inferior for speed. In a recent experiment by Brownell and Moser (4) the decomposition (meaning) method was compared with other rule methods in learning the subtraction facts. Significant differences were found for the decomposition method by rational means. Division was the arithmetic process studied by Anderson (1) who found that the meaning method was effective in transfer of training in this process. An experiment by Howard (5) checked short and long range retention for the meaning and rule methods in the area of fractions. He found that the rule method was superior during the semester but that the meaning method was significantly superior after the period of retention.

This summary of the research on the meaning and rule methods shows that very little research has been done to check on the effectiveness of the meaning method for these specific problems: 1. the areas of arithmetic such as the basic or fundamental process, fractions, decimals and percentage. 2. certain degrees of arithmetic complexity such as definition, simple size

relationships and complex analysis and, 3. the retention of the material after the initial experiment has been completed. Several questions arise from these problems. Is it possible that the meaning method is more valuable for one area of arithmetic than another? And if so, what are these areas of maximum effectiveness? Is the meaning method superior for learning in the simple or complex analysis of problems in arithmetic? What is the effect of learning after the experiment has been completed? Do the students only make tentative gains or are these advances in learning of a permanent nature? Obtaining the answers to these questions was the impetus behind the Los Angeles Study.

The Experiment

At the beginning of the semester incoming B7 students in five junior high schools in the Los Angeles City School District were given two tests: The California Arithmetic Test and the Meaning Test. The first was selected since it was a well known standardized test involving computation in arithmetic. The second test was devised for this experiment to measure the degree of understanding of arithmetic. These tests were given at the beginning of the semester, the end of the semester and after the summer vacation. The test results were used to examine the difference in gains between the test scores during the semester and after the period of retention.

This experiment differs from others as it tested all areas of arithmetic. Previous investigations only considered one area of arithmetic or one segment of arithmetic such as the 100 addition or 100 subtraction facts. Therefore, the composite scores for all learning during the semester and after the period of retention as well as the scores for the areas of arithmetic were obtained.

In order to establish the method of instruction used by each teacher, the experimenter made six to eight observations in the classrooms of eighteen instructors during the semester. These classes represented a cross-section of the population of Los Angeles. Data concerning their lectures and method of instruction were obtained by observation and recorded. No attempt was made to suggest any procedure of instruction. This feature was different from other experiments in which the method to be taught was specified or implied. In this way normal classroom techniques could be observed without any alteration of the actual instruction. This procedure could lead to more accurate classification.

After the data was collected it was analyzed and the instructors were rated for meaning and rule content in five different categories: 1. lectures in class 2. type of classwork 3. type of text 4. kind of tests, and, 5. an independent evaluation by means of a supervisor's rating. An instructor was rated as high in each category if he emphasized the meaning method. He was rated as low if he emphasized the rules without making them meaningful. He was given a medium rating if he used a combination of both methods. The results were compiled and placed along a continuum. Eight instructors were selected at the two extremes: four for the meaning method and four for the rule method. These instructors were the most representative for each method.

The students whose data were included in this experiment were those taught by the eight instructors. This group of over six hundred students from a total of fifteen classes served as the reservoir from which individuals were matched on the basis of Otis I.Q.'s and scores of the Meaning Test (matching was done on the basis of a maximum of a two point difference in test score). In cases of duplication the results of the California Arithmetic Test were used to equate the pairs. A total of one hundred and eighty matched pairs was involved in the tests during the semester, and a total of ninety-five pairs was involved for the test outcomes after the summer vacation.

In this experiment not only were the total test scores for both groups compared but also certain finer subdivisions were studies such as areas of arithmetic, degrees of complexity of arithmetic and I.Q. levels. These subdivisions are as follows:

1. AREAS OF ARITHMETIC
   a. California Arithmetic Test—basic processes (addition, subtraction, multiplication, and division), fractions, decimals, percentage, mixed problems (decimals and fractions) and measurement.
   b. Meaning Test—basic processes, number systems, decimals, fractions, percentage and measurement
2. DEGREES OF ARITHMETIC COMPLEXITY (for the Meaning Test only)
   a. 1st degree—definition
   b. 2nd degree—simple size relationships
   c. 3rd degree—complex analysis
3. I.Q. LEVELS
   a. Low (70–89)
   b. Average (90–109)
   High (110–140)

Findings of the Study

1. The Meaning Test showed a near significant difference (10 per cent level) in favor of the meaning method at the end of the retention period.

2. The California Arithmetic Test produced a near significant gain (10 percent level) in favor of the meaning method at the end of the semester and a significant difference (5 per cent level) at the end of the retention period.

3. The Meaning Test showed a significant difference in favor of the rule group in the area of measurement at the end of the semester.

4. The California Arithmetic Test indicated significant differences in favor of the meaning method for the areas of mixed problems and decimals at the end of the semester and for the area of fractions after the vacation. A significant difference in favor of the rule group was noted in the area of measurement at the end of the semester.

5. The Meaning Test revealed significant differences in favor of the meaning method for the highest degree of arithmetic complexity (complex analysis) for the period of retention.

6. The Meaning Test showed significant differences in favor of the rule method for the Low I.Q. group for the end of the semester only. However, statistical analysis showed that this difference may have been the result of a bilingual handicap which entered after the experiment had begun. Recent research (2) shows that students with a bilingual handicap do significantly poorer in scholastic endeavors that those students without this deficiency. The imbalance in the ratio of bilingual students in the meaning and rule sections of the Low I.Q. group was brought about by the failure of one instructor to administer a test at the proper time. This caused a larger percentage of bilingual students to be found in the Low I.Q. group taught by the meaning method. Prior to this time the students in both sections of the Low I.Q. group were approximately equal in ratio. The study also showed that the average and high I.Q. groups produced significant differences favoring the meaning method for the period of retention.

7. The California Arithmetic Test showed significant differences in favor of the rule group for the low I.Q. group during the semester. However, this difference again may be attributed to the bilingual factor suggested above. The high and average I.Q. groups produced significant gains favoring the meaning method for both the period during the semester and for the period of retention.

Conclusions

1. The meaning method was more effective for the area of computation of fractions. The areas of decimals and percentage showed gains favoring the meaning method but only during the semester. The rule method was superior for the area of measurement but only during the semester.

2. The meaning method was more effective in establishing retention in the processes of computation as well as for the understanding of the principles of arithmetic.

3. The meaning method was more effective for the comprehension of complex analysis in arithmetic indicating a potential superiority for difficult concepts.

4. The meaning method was more effective for the average and high I.Q. groups. The rule method seemed to be more effective for the low I.Q. groups but the results are open to doubt due to a bilingual factor.

Recommendations

1. Continued research is necessary to establish the effectiveness of the meaning method in all areas of arithmetic. Repetition of experiments is necessary to validate the conclusions of the studies in areas of arithmetic.

2. Better design of experiments is necessary to examine the role of the meaning method for the comprehension of complex processes. Further verification of this trend could lead to important implications for arithmetic learning.

3. A complete investigation should be made to check the value of the meaning method and its effect on increased retention. Such a study could provide evidence for a greater mastery of arithmetic by the students through better teaching techniques.

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EDITOR'S NOTE. The Los Angeles Study has given us some tentative results which Mr. Miller suggests should be further investigated in other research with perhaps a different design and control. It is very difficult to compare two methods in terms of statistical results because of the many factors which enter into a learning situation. This study is a very good beginning. It is good in that it covered the total arithmetic program over a semester of time and measured retention after a summer of potential forgetting. Too frequently it has been assumed that the results of many small experiments can be added for a general result or conclusion.

The Meaning Method, as a mode of learning, is not as discrete as the Rule Method because there are many ways to develop meanings and understandings. We need some good research to show how this can be done most effectively for both short and long term results. The measurement of meanings and understandings is not easy. The editor feels that most of our tests in the area of understanding should be fortified with the judgment of a competent teacher.

We are thankful to Mr. Miller and the Los Angeles Study for opening this important field of investigation.

Pre-First Grade Arithmetic

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CHILDREN'S EARLIEST EXPERIENCES with number are pleasurable ones. Before they use words or understand many words, children display responses to the *manyness* and *numerousness* of objects or events in their world-of-action and their worlds of seeing, hearing, touching, and moving. Their sensory reactions are so inextricably fused with their movements and manipulations that they are best characterized as sensori-motor activities.

Simple performances such as assembling, piling, accumulating or collecting of objects that are alike, such as their toys, pebbles, or pinecones, delight children repeatedly; especially when they can lay hands on any such collection and throw the objects gleefully around. Well known to all who observe children is their intense enjoyment that accompanies rhythmical movements of various kinds, swinging of the body or clapping of the hands, and the inner urge to synchronize such movements with the rhythmical songs of the mother or the child's own articulations.

In early verbalizations, children often repeat words "more, more," or "one, one," while touching or pointing to one after another of their toys or objects, obviously meaning "another of the same." Still later in his development, the child can sometimes be heard reciting "numbers" while he busily engages in some activity such as lining up cars or blocks. "One, two, four, seven, ten," he recites as he assigns his possessions with a "number" without actually understanding "how many" he really has.

At this stage, number is language, and it is evident that the child becomes readily conversant with numbers and able to put them to use in a fairly adequate fashion, without understanding all of the subtle implications of number. For instance, the child has many experiences with nursery rhymes that include number names in content over and over again. (Rub-a-dub-dub, Three men in a tub," or "Gregory Griggs, Gregory Griggs, Had twenty-seven different wigs.") The old story-time favorites of "The Three Bears" and "The Three Billy Goats Gruff" have been retold to the child without ever seeming to lose any of their flavor. Also such songs as "Five Little Chicadees," and "Ten Little Indians" have echoed throughout many a home.

When the Child Enters School

But what happens to these pleasant experiences and activities when the child's world of number takes on the formal aspect of the school? The tendency has been not to capitalize, enrich, and broaden all of these important and early numerical responses, but to feed the child with all types of "concrete" materials; and hope that while he tries to digest them by looking at them or handling them, he will eventually learn the concept and meaning of number! The child's experiences in the kindergarten should not be of a different kind than those he enjoyed previous to entering school, but should differ rather in degree. The order and manner in which these experiences are arranged for the child are important factors in developing and enriching the number concept for the child.

Researchers have repeatedly pleaded that teachers should note and allow for the knowledge of number possessed by the individual child before entering school life. The recognition of the child's number de-

velopment can be easily observed and utilized by the teacher through planning of pre-counting activities. In planning for pre-counting experiences, it is essential to adjust them to the age, to the mental ability and to the phase of development of the child. A free expression of their rhythmic tendencies is desirable as a basis for their earliest notions of number, and exercises appealing to the visual, auditory, and kinesthetic senses are especially recommended.

Kindergarten children enjoy all sorts of rhythmical exercises, especially those that are centered around musical activities. A musical and rhythmic unit centered around the theme of *Circus Day* holds much appeal for children. For example, the circus tent is erected by "hammering" the stakes to four beats to a measure with musical accompaniment. The swinging of the trapeze artist calls for three beats to a measure: the tight-rope-walker proceeds at a slow four beat; while the jesting monkey performs to a faster six beat—and so on. The teacher and children both can use their imaginations and creative abilities to include more rhythms to extend the unit of this type, or create other activities fashioned after the circus theme.

Skipping, hopping, galloping, swaying, or jumping to definite rhythmic beats, as well as clapping of the hands, stamping of the feet, beating of the drum, clashing of the cymbals, or tingling of the triangles in direct imitation of the teacher, or in response to music are also other rhythmic activities that help create for the child a feeling for number.

One-to-One Correspondence

Not only should the child's native ability to sense rhythm be utilized as one of the basis for a sound formation of the concept of number or precounting experiences, but his ability to perform other serial-motor activites can also be channeled in a meaningful fashion. In guiding the child to perform serial-motor activities, our main purpose is to help the child establish for himself the basic principle of counting, namely a ONE-TO-ONE CORRESPONDENCE, that is the matching of items in a group with a standard series of things or symbols.

One can determine, for instance, whether there are in a classroom just as many chairs as children. All one has to do is assign a chair to each child. Thus, one has formed pairs, each of which consists of a chair and a child. The chair and the child are matched in a one-to-one correspondence, that is a chair corresponds to one child, and one child to a chair. If there are no vacant chairs and no children unprovided for, the two sets are similar or equivalent, they have the same number.

The child also performs serial-motor activities of a one-to-one correspondence in nature while he distributes milk cartons to each child at milk time, while he passes out paper towels to each child as he washes his hands, while he hands out boxes of crayons, pieces of construction paper or boxes of paints at art time. Thus, the child has been matching, pairing, and corresponding one series of objects against another series of objects in a functional situation.

Activities for performing serial-motor activities of a one-to-one correspondence can be made alluring for the child by various action situations presented by the teacher. Children are usually fascinated by early rustic ways of life, and can be aroused by appealing to their imagination by presenting the problem that someone else "does not know how to count." Such problems about the "ways that Indians, who could not count, were yet able to keep track of their ponies," and other ways primitive man handled quantitative situations offer a wealth of interesting and stimulating materials that can be easily adapted to the kindergarten level.

History tells us that primitive man pursued "numeration without number" primarily to keep track of his possessions, to guard against losses or robberies, or even to compare his possessions with those of

his fellow tribesman. Any set of small objects like sticks, pebbles, splints, shells, kernels and later fingers, were used as an ideal auxiliary means for checking his possessions. The auxiliary set had no name, yet the items were just as exacting as numbers for showing him "that many" or calling his attention to any of his losses or gains. To this day the Apache Indians carry around with them a bag of pebbles which correspond one by one to their herd of ponies. As they let their ponies file through a narrow passage in front of them, they put one pebble for a pony into their bag, only to reverse the procedure when the ponies come home from their grazing fields.

Once he had conceived the idea of conveying to others the size of his possessions by means of matching and pairing, the next step for primitive man was to record what he had before him for practical use or for posterity. A variety of symbols was devised still in keeping with the one-to-one correspondence, and of these symbols the stroke or tally notation is still very lucid in conveying meaning. For each possession the stroke of / was recorded, and / / / / / conveyed the meaning of just "that many" ponies, "that many" bushels of wheat, and so forth. It is this old *natural* way of recording number that becomes the most practical method of numeration of which we should introduce the child before we teach him to use our notation.

The child while playing a vigorous game of cowboys or Indians might be very interested in knowing "how many" Indians were shot down, or "how many" cowboys were saved in this battle. He could easily be led to find out by using a stroke notation for each Indian shot down or each cowboy left on his horse. The picture of the ensuing battle becomes more meaningful for him, more interesting, and more permanent, because *now* he knows that there were just "that many" shot down or left standing, and can easily make comparisons for future battle engagements. Many more functional situations of establishing a one-to-one correspondence, and recording the correlation without number words can be devised by the teacher to meet the needs of the group. The children should also be stimulated into inventing some recording signs of their own besides the stroke notation.

Fingers are Useful

We cannot help but turn to primitive man once more in helping us in our next pre-counting experiences, that of teaching the child to tally on his fingers. When primitive man used his fingers as tallies, each finger not only had its individual character, but could be easily located in a fixed place in a series. As he started to tally with his fingers, primitive man soon discovered that the *name* for each finger place could be transferred to the objects being tallied; and by halting at a finger, he could see just how far he had arrived in the series. The tallied objects not only were distinct, but also *different* because of their position or place in the series. The names given the individual fingers are probably the roots of our present number words. The derivation of the word "five" from hand—since five finishes the tallying on one hand—is still recognizable. The instrumentality of the fingers and their fixed spatial relations of *middle*, *between*, *before*, and *after* was the key to the progress from tallying to counting.

The fingers are certainly ideal instruments for helping the child arrive at a meaningful concept because he is familiar with many of their characteristics long *before* he learns to use them in specific experiences as a reference series, and because their movements appeal to more than just one of his senses. Tallying on fingers arouses visual, tactual, and kinesthetic sensations, all helping to construct a spacial model from which to infer relations between the elements of the series. The child should become so thoroughly familiar with the positions of his fingers, that he

can refer to each not only by our customary descriptive terms, like ring finger, but also by the descriptions that accompany fingering in primitive languages. An intuitive phrase like the primitives' "go to the other hand" for "six" implies numerical value due to definiteness in regard to positional meaning that cannot be surpassed; while an insipid term like "six," at least for the uninitiated, is devoid of all meaning. Meaning of number words like meaning of all words—is established in active behavior of the child, since no one creates ideas in the child's mind but the child himself.

There is a great deal of significant material available from which the teacher can glean expressions that are conducive to further development, in regard to each particular location of the fingers on the hand. The teacher is free to invent such phrases of her own choosing; however, she will find helpful phrases borrowed from primitives, such as the Dene-Dindje or Zuni Indians.

The Dene-Dindje Indians begin with the little finger, *1*, "the end is bent": *2*, "it is bent again," then to the middle finger, *3*, "the middle is bent": touching the index finger they say *4*, "one is still left," and the thumb, *5*, "it is through on my hand." Similarly the series of the Zunis runs like this: *1*, "taken to start with"; *2*, "put down together with"; *3*, "the equally dividing finger"; *4*, "all the fingers all but done with"; *5*, "the notched off."

Now that the teacher has some significant materials at her disposal, she may ask, where does the child begin, on his left hand or right? How shall the child hold his hands, palm facing the body or the other way around? In teaching "fingering" the teacher must not stand in front of the children, but beside them, since the children are apt to be confused when their right hand confronts the teacher's left and vice versa. In other words, they must be able to see a direct, not a mirror image of their own hands in following the teacher's instructions concerning the movements on the fingers.

The child holds the palm of his hand toward his face. The little finger of the left hand becomes the number 1 finger, ring finger becomes the number 2 finger, middle finger becomes number 3, pointer finger becomes number 4, and the thumb 5. For number 6 we go to the other hand, and the little finger becomes 6, ring finger 7, middle finger 8, pointer number 9, and thumb 10. If left-handed, a child may reverse the process when he is not following the "fingering" of the right handed person. This does not make any difference as the order and relationship of the fingers will be the same to him.

Thus through the urge to make use of the mobility of his hands, the child can be instructed to see definite associations between spatial relations and numerical expression. He will be able to recognize, to differentiate, to choose and name on command individual fingers on his own hand or on another person's. He will be able to visualize the great many relations between the members of the series. When the scheme has been firmly established in his mind, he will be able to identify a position without counting up to it. For instance, finger number 3 is the middle finger, finger number 4 is the one before the last, or number 8 is the middle finger on the other hand, and the like. The child will gain freedom to recognize a certain position from various vantage points: he will develop the same ease in determining which finger comes *before* another finger as he has in finding out which comes *after* another.

As we have seen, each finger indicates a specific position on the hand, but it also indicates the number of fingers touched or moved in reaching this position. In tallying on the fingers, finger number 3 is taken to define a group of three by virtue of the steps 1, 2, and 3 needed to arrive at this point. In the beginning therefore, when the child is asked to show 3 fingers, he should show fingers numbers 1, 2, and

3, rather than 3 odd fingers. This continuous process from one to the next brings home to the child that each suceeding number is larger by one than the one that comes before. The series of numbers will no longer be a mere string of words, but will become a well-structured sequence of numbers generated by the recurrence of the relation of *plus 1*.

However, when the child is no longer limited in showing for instance, a three (on fingers 1, 2, and 3) but discovers that any 3 fingers will represent number 3, and that numbers denote collections of things as well as position, his idea of number will now be based upon the unchangeable abstract series of numbers rather than on any fixed pattern of fingers or objects. "Fingering" has changed to counting, and number words alone suffice to evoke the meaning first established on the fingers.

Summary

Thus, in planning pre-first grade arithmetic learnings, we should make use of the child's natural endowments for learning the meaning and concept of number: one, the child's sense of rhythm; two, his ability to perform serial-motor activities; and three, his ability to learn the spatial image of his hand. If the child is led in his pre-school days to follow the path of nature, he will be less prone later to depend on meaningless counting on fingers—the very opposite of knowledge gained by learning to associate the structure of his hand with numerical relations.

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EDITOR'S NOTE. It is remarkable how many ideas and concepts that children have formed before they enter school. It is also rather revealing to inquire into these apparent concepts and to discover how many of them are very superficial and exist only at the "word stage." Dr. Riess has suggested two interesting approaches to a natural development of number sense and number understanding. Rhythm gives both a physical and a mental association with number and sequence. Finger exercises seem to be very natural with all peoples. Some teacher will say that Dr. Riess spends too much time with "fingers and numbers." She is careful that genuine understanding develops and that children move slowly but surely to abstract ideas. Note how different this approach is from the older method of beginning with rote counting and with number symbols. Always, we must bear in mind that each child must develop his own understandings and concepts.

Achievement by Pupils Entering the First Grade

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WHEN ASKED "how many inches in a foot?" the five year old child, his first day in first grade, removed his shoe and counted his toes. The question again was repeated to him. Very arrogantly he eyed the interviewer and said, "Don't you know people have toes and not inches in their feet." Reluctantly the interviewer proceeded to ask the next logical question. "Johnny, how many feet are there in a yard?" To this question John hesitated for one moment. Then appearing to be a little baffled he could be heard in almost a whisper, speaking as if to himself, "Joey—Jimmy—Tommy—Peter that makes four and me is five, five and five more is ten." "I know," he yelled as pleased as punch, there are ten feet in a yard." The interviewer gave quite a perplexed look, to which John grinned with all the satisfaction a five year old could possess, and said "You didn't think I knew, did you?" When Johnny was asked how he arrived at this number he said, "Simple, all you got to do is add both feet of all the boys that play in your backyard."

Daily children are confronted with arithmetical facts. How do they react to them? The child in the above interview was not familiar with the meanings of feet and yards, as we know them, but was able to compute five and five more. He remembered that children have two feet each which was a factor in arriving at ten. Do we meet the needs of the individual children in our classrooms today? Do we attempt to discover how much understanding they have in arithmetic? Do we adjust our teaching to meet their needs, or do we adjust their needs to meet our teaching? These are typical questions which are asked constantly. The answers can only be arrived at after we conscientiously attempt to discover what children know when they come to school, then adjust the curriculum to develop understandings from this point.

The problem is not only disturbing to the classroom teacher, but to parents, industrial leaders, college professors, etc. The cry has been that children are not gaining the understandings in arithmetic. Arithmetic is not being taught meaningfully.

In September, 1953, during the first two weeks of school, all the children entering the first grade for the first time in a school in upstate New York were tested by the same examiner in a room set up for the testing program.* The object of the test was to determine how much arithmetic boys and girls knew when they entered school. An assumption was made that the curriculum could be adjusted later to meet the needs of the children entering first grade.

In order to test all the various phases of arithmetic, it became necessary to make a test with 14 subheadings. The time for this would be approximately thirty minutes per pupil, with the test administered orally. The games were made varied and interesting to hold the interest of the child for such a long period of time. Many of the objects used in the test were blocks and pegs which children had used in kindergarten. A number of these were placed in closed boxes to keep the children interested and not to sway their attention when working with one test.

A careful study had been made of all existing tests in this field on this level. An

* This survey was made in a single school in a city of 100,000 population and involved 70 children. The clientelle of the school is considered average.

assimilation was made of all the types of items which are most helpful in a test of this nature. The test was then developed with the items placed according to their level of difficulty.

Test I—Rote Counting

a. Counting by 1's
b. Counting by 10's
c. Counting by 5's

In the test of rote counting, the child was merely asked to count by 1's, as 1-2-3, by 10's, and by 5's. The results showed the average ability to count to 29.69 by 1's. This is a result that should be viewed carefully. A number of our workbooks for the first grade advocate spending weeks on counting from 1–10, however a number of children can count beyond this their first few days in school. If this is true then some adjustment should be made immediately. Children were less able to count by 10's. Only 10% could count to 100, while only 4 of the children tested could count to 100 by 5's. This is a revelation since we do not attempt this until the later part of the first year or during the second grade. If children can count groups at the beginning of the year, maybe they are ready for instruction in group work.

Test II—Rational Counting

a. Counting by 1's
   Scatter 100 discs on a table before the child.
b. Counting by 10's
   Place ten groups of tongue depressors, with ten in each group, in front of child. Take one bundle apart and show that it contains 10.
c. Counting by 5's
   Place 20 groups of colored pegs, with five in each group, in front of child. Take one bundle apart and show that it contains 5.

In the rational counting test a measure was being made of the child's ability to note the one-one correspondence of numbers with objects. In counting by one's the children were able to count on the average to 29.77, which was slightly higher than the rote counting. Although the children were not familiar with counting by 5's and 10's, they seemed to be able to get the idea in rational counting and made some attempt at it.

Test 3—Reproducing

Ten colored blocks were placed on the table in front of the child. He was asked to give the examiner 5, 3, 6, 4, 7, 2, 8, 1, 9, and 10, of the blocks. The test showed that over 75% of all the children tested could reproduce numbers to 10 by selecting the correct number of blocks when they enter first grade. Every child tested scored perfectly on the numbers 1, 2, and 3. An interesting observation made was that more children could reproduce 10 than 9. Perhaps they are more familiar with groups of 10.

Test 4—Identifying

This involves a more difficult task than reproducing. The child is shown a specific number of beads and asked to tell how many there are. The order in which they were asked was 5, 3, 6, 4, 7, 2, 8, 1, 9, 10. This, as had been expected, was more difficult. Fewer children were able to identify the correct numbers of beads. However, 59.9% of the children could identify all the numbers up to 10. If over half the children can do this at the beginning of the grade, there appears a need for grouping in arithmetic.

Test 5—Combining with Objects

Small blocks were used for this test. The test was given in the following manner. The examiner would say, "If I give you 2 blocks (as this was said, 2 blocks were placed on the table, and covered with hand), and then I give you 2 more, (cover these with hand), how many blocks would you have?" The following combinations were used.

5 and 1	6 and 2
2 and 4	4 and 1
9 and 1	1 and 2
2 and 3	3 and 2
3 and 7	

The most common know addition fact was 2 plus 2. Three-fourths of the children tested knew this combination. Closely following this was the combination 1 plus 2, which 74% of the children were able to compute. It appears that in any combination when one was added to another number it was easy to get the answer. This probably implies that children understand the sequence relationship of numbers. They understand that any number following another number in counting, is one more than the number preceding it. The most difficult combination was 3 plus 7. However; in every other combination over 34% of the children could provide an answer. Some of these combinations are not taught until the second half of the first grade. Many schools will not teach any combination with answers more than ten in the first grade. Some adjustment will need to be made, if the truth is that children can combine to ten when they first enter first grade.

Test 6—Combining without Objects

No objects were used for this game. The directions given were to tell how many things Mother had brought us from the store. In investigating Mother's imaginary bag we found she brought many things which we already had at home. When these things were combined how much would we have all together. The combinations were the same as those used in Test 5. Some of the items which Mother brought were boats, balloons, oranges, bubble gums, crayons, cherries, peppermints, popsicles, airplanes, teddy bears. Items used were those which the author felt would appeal to boys and girls. We must keep in mind that the children had no objects to manipulate or visualize in this test. The child had the task of transposing the abstract number in his mind to something more concrete and then combine the two figures. The results of the test were close to those of Test 5. The test showed that 14.25% of all the children tested could compute all the combinations. The most difficult combination also was 7 plus 3. Over 26% or ¼ of all the children could compute all the other combinations with 2 plus 6 being second in difficulty.

Test 7—Separating with Objects

Small beads were used for this test. The directions for this were "I will give you 2 beads (put 2 beads before the child) and when I take one back (remove 1 bead) how many beads do you have left? The following combinations were used:

2—1
5—2
3—1
4—3
6—5
8—4
6—1
7—3
9—2
5—3

The hardest combination to subtract was 2 from 9, and the easiest was 1 from 3. Fifty per cent of the children were able to do most of the subtraction facts given. This may be due to a number of reasons. However, if they can do this much at the beginning, they may be ready for instruction in this phase of numbers.

Test 8—Separating without Objects

This test was given without any objects. It resembled Test 6 in scope. The children were told that they were to pretend that they were going to have a party. After giving some of the items away to the guests, how many would be left? The imaginary items used were pears, party hats, dolls, marbles, blocks, packages of gum, balls, books, drums, and kittens.

As you can recall the test items (see Test 7) were listed in order of difficulty. This test was more difficult, although each combination received correct responses from some of the children. The most difficult was 2 from 9, which 14.3% of the

children knew. This test had the same type of difficulty as Test 6, which was to transpose the imaginary items to numbers in the head, and then obtain the correct response. The easiest combination was 1 from 2, which 91% of the children were able to do. Although this test was rather difficult, the children seemed eager to continue and made attempts to discover the correct answers. No one was anxious to discontinue and was interested in going on.

Test 9—Fractions

A felt board was used to test fractions. Three colored circles of the same size were divided into fourths, thirds, and halves respectively and placed on the felt board. The child was told to pretend that these circles represented pies. "If someone asked you (the child) for $\frac{1}{2}$ a pie, which piece would you give him?" The same was done with $\frac{1}{4}$ and $\frac{1}{3}$. The results of this test reveal that 50% of the children knew $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$. This concept of fractions is considered to be difficult and is not introduced into the grades for a while. However, if the children come to school with the ability to identify these fractions, they may be ready to learn a little more about fractions. The most difficult of the fractions was $\frac{1}{4}$ which 50% were able to recognize. The next in difficulty was $\frac{1}{3}$ which 51.4% were able to recognize. The easiest was $\frac{1}{2}$ and 77.5% were able to recognize this. Children have many occasions to divide objects in half; half a candy bar, half an orange, and half a glass of water. They have recognized the connection between half and something divided into two parts. Perhaps they don't realize that half means two equal parts, at this point, but they connect half with two, which is a beginning understanding. The same is true of $\frac{1}{4}$ and $\frac{1}{3}$.

Test 10—Facts

There are certain facts in arithmetic which children learn sometime. When are the children ready to learn these facts? Some of them have been included in this test. The fact that the largest percentage of children knew was the number of eggs in a dozen. The children who knew were asked how they knew the answer. A few of them responded that their mothers had egg containers. They said they had counted the number of spaces in it. Almost 10% of the children knew the number of feet in a yard. The next fact according to correct responses was the number of days in a week. Closely connected with this and receiving almost the same amount of correct responses was the number of days a week you come to school. Following this was the number of inches in a foot. Only 4% of all the children tested knew the number of hours in a day. It is interesting to note that more boys are cognizant of these number facts in arithmetic than girls are.

Test 11—Recognition of Symbols

We are under the assumption that children enter school unable to read and write. There are many exceptions to this rule, but it is an assumption which applies to the majority of situations. However, the next test was for the child to recognize the symbols that represent the numbers 1–10. These numbers were placed in random order on a 5 by 8 inch card. The children were asked to point to the number which represented 3–7–5–2–6–8–4–9–1–10. More children were able to recognize 3 than any other number. The number 7 was recognized by 60.15% of the children. The same number of children who recognized the number 2 also knew the number 8. About 43% of all the children recognized all the numbers through 10. The most difficult number to recognize was 6, which 43% of the children knew. Next to this was 9, which 44% knew. This combination is often difficult for children to recognize and some children continue in later months to write 6 when they mean 9 and vice versa.

Test 12—Judgments

Certain judgments are made daily by all of us. When do these have their beginnings? How do storekeepers estimate a pound of grapes before placing them on the scale? How can a dressmaker estimate 3 yards before measuring it? How can a baker estimate a quart of liquid? A few judgments were asked for. A quart of milk is a very familiar item in the home. Each child has been exposed to it by the time he enters school. However, when presented with it and asked to identify it, only 59.9% of the children were able to do so. Practically the same number of children were familiar with half a pint which is not as frequently used as the quart. A little over 5% of the children recognized a pint which can be readily understood. The pint bottle is not often seen by children in their everyday experiences. In the questions concerning linear measure more children recognized the yard than the foot and inch. Forty-four percent of the children could estimate which of four boxes contained a pound of candy. Twenty-nine per cent could recognize which of four lengths was an inch, while 21% could recognize a foot. The results of this test may show that children have heard these terms and are familiar with them.

Test 13—Money

We realize that children have experiences with money long before they come to school. They are sent to grocery stores, are given allowances, have piggy banks, and other such means of handling money. Because of this a section on money was included. Over 75% of the children tested knew which is more—3 pennies or a nickel. Next to this in order was, "which is the same as a dime—6 pennies or 10 pennies?," which 71.3% knew. Fifty-seven per cent knew, "which is more—12 pennies or a dime?"; while 56% knew, "which buys more, a nickel or a dime?" Fifty-three per cent knew, "which is the same as a nickel —5 pennies or 8?" The next in order was the number of pennies in a nickel which 35.75% knew and following this was the number of pennies in a dime which 25.5% knew. The most difficult concept here was the number of dimes in a dollar. No girl was able to answer this and only 14% of the boys knew the answer. Perhaps boys have more experience with more money earlier in life than girls do.

Test 14—Time

This test was sub-divided into two areas: identifying and reproducing. The results show that 41% of the children could reproduce the number 2, and 35.5% of them could reproduce the number 4. The number 12 was identified by 25.6% of the children while 21.3% could identify 9. The results show that children can reproduce time more easily than they can identify it. However, most people teach children how to identify first, which may be harder. The hour 9 was chosen because it is the time school begins and was felt that children may have become familiar with it. In the same reasoning 12 was used because it is lunch time. Two and four were picked at random. However, more children were familiar with the two numbers which were picked at random.

Conclusions

From the results of this test some conclusions can be drawn. Children have many uses for arithmetic prior to entering school. This is evidenced by the amount of knowledge they possess when they enter school. They employ numbers in games, addresses, phone numbers, television, etc. Numbers are a part of their everyday experience. Consequently it is deemed advisable to begin the right kind of arithmetical instruction in the first grade.

The incidental approach to the teaching of arithmetic does not guarantee that the children will learn all the skills and concepts needed to better understand arithmetic. A systematic approach would provide for an orderly development of con-

cepts. Since the development of skills takes time the teacher should provide varied experiences to direct children to discover higher and more mature procedures. The curriculum for teaching arithmetic in the primary grades should be based on the amount of arithmetic children know when they come to school.

The first question to consider is the ability of the children to learn arithmetic. We may assume that with proper instruction more arithmetic can be taught. It would be an insult to the intelligence of most of them to spend a number of weeks teaching what they already know.

Another question to be considered is the need of the first grade children to learn arithmetic. Since the children have "incidentally" gained so many skills before entering school, they must have had a need motivating them to learn. There are many "social" uses prevalent in young children's experiences for arithmetic. A number of games intended for five and six-year-olds demand some knowledge of arithmetic to be played successfully. The desire to use the telephone promotes the recognition of numbers in many cases. Judgment words as pound, dozen, foot, etc. are used in store and play activities. According to these facts one may note that children do need arithmetic. If for no other reason, some may need it to be accepted by their peers.

It is not possible to outline a good program from the conclusions of this study alone. However, it may be seen that a revision in the present curriculum should be made to provide for a systematic program which is based on understandings and meanings. We owe this to the children who enter school with some background and a desire to learn.

EDITOR'S NOTE. Miss Priore not only reports the results of her survey of arithmetic learnings possessed by entering first grade pupils but she also raises some of the implications faced by a school when pupils come with a background beyond that normally expected and specified in generalized courses of study. But the range of achievement at this level is probably less than at any later level. At all grades we are faced with the same problem. It is not enough to recognize the problem, it is one that should be investigated in some detail by each teacher as she faces a new group of pupils. Usually it is not possible to sit down with individual pupils and make a detailed analysis of previous learnings. Many teachers begin the year with a review and inventory which supplements the reports sent on to them by the previous teacher. Then group work is in order. There are many good plans for grouping for specific purposes. The better schools seem, at the moment, to be using some form of group work coupled with whole class participation in the major class development.

Miss Priore's inventory, while not complete in terms of depth of understanding, is very suggestive of the type of investigation that more schools should be doing. After the inventory the real work of organization and instruction must begin.

Composition and Arithmetic

When one is writing a letter or a composition it is usually possible to use a substitute word or expression when one is uncertain of the spelling of a particular word or of the acceptability of a construction. Likewise, in arithmetic one can frequently find a substitute method for solving a problem. Usually there is no one best method for all people. There may be a more direct method in arithmetic just as there is in using the English language but choice of method is often a matter of opinion. In English usage one form may be more acceptable but the final test is one of transference of thought. So too with arithmetic, the final test is whether the answer is correct. "Them haint yourn," conveys thought to certain people but is not school-book acceptable. To find the cost of $\frac{3}{4}$ pound at 49 cents per pound the clerk performed: $49+1=50$; $\frac{1}{2}$ of $50=25$; $\frac{1}{2}$ of $25=13$; $25+13=38$; $38-1$ (amt. originally added) $=37$ cents. The school-book method yields: $\frac{3}{4}\times 49=36\frac{3}{4}$ or 37 cents.

Introducing Our Numbering System in the Primary Grades*

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There are many objectives for the teaching of arithmetic in the primary grades. One of the most basic of these objectives is that of helping children to understand the place value principle of our number system. It is in the early grades that foundations of understanding are developed, but it should be pointed out that not until the student has studied exponents in high school is it possible for him to consummate these understandings in the richest possible way. It is the purpose of this article to discuss some of the fundamental approaches that can be used successfully in the primary grades.

Pre-Number System Learnings

One of the first things the teacher should make sure of is that the pupil has a good understanding of numbers from one to ten. This should include such abilities as being able to count to ten with understanding, being able to read and write numbers from one to ten, being able to make comparisons involving these numbers, and being able to analyze numbers from one to ten so that eight, for example, may be thought of as being composed of a group of six and a group of two, or a group of five and a group of three, etc. This last point involves the important concept of grouping which should be introduced by experiences surrounding such questions as which group is larger? Which group is smaller? How much bigger is this group than that group?

As in all good teaching, especially at the elementary level, the general pattern should proceed from the concrete stage to the abstract stage as gradually as is feasible. Usually the teacher should attempt to include two intermediate steps in the form of pictures and semi-abstract representations such as dots, crosses, circles, etc.

The group concept which was mentioned previously is an extremely important prerequisite concept to understanding our number system. As indicated, the concept should be developed through experiencing activities calling for the analysis of a number such as six. For example, a group of six children may be composed of the subgroups of two girls and four boys, or of five girls and one boy. Children should be encouraged to look for the various subgrouping possibilities. As an aid in developing the concrete and semi-concrete aspects of grouping, the teacher should have available, collections of various toys, erasers, pencils, books, bottle caps, clothes pins, jacks, checkers, etc.

There are also available many simple visual aids which will facilitate the work of the teacher in developing group concepts. Some of the more useful of these are the bead frame, the abacus, the flannel board, the magnetic board, a coat hanger from which one may suspend varying numbers of clothes pins, and a number fence.[1] (See diagram 1.)

* A digest of a talk given at the New Jersey Education Association Mathematics Conference, October 1956.

[1] A number fence can be constructed very easily from a piece of wood about two feet long, one or two inches thick and about two to four inches wide. Twenty evenly spaced holes should be drilled and then a saw cut about a half inch deep should be made between each of the holes across the width of the board.

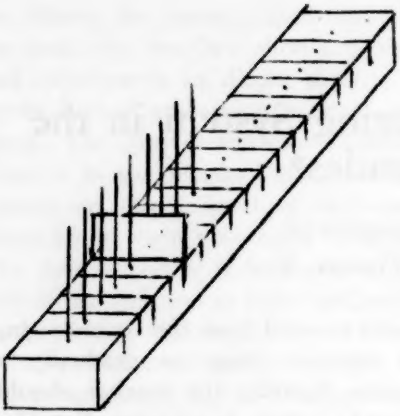

DIAGRAM 1 (Number fence). Showing that a group of nine is composed of a group of five and a group of four.

Beginning the Place-Value Idea

To develop the number concepts from eleven to twenty it is merely necessary to extend the idea of grouping so that one of the subgroups will be ten and the other subgroup will be whatever is necessary to yield the number under consideration. For example, eleven should be emphasized as a group of ten and one more. At this point 11 should be written on the board and it should be emphasized that the digit "1" on the left indicates one group of ten, and the digit "1" on the right indicates one object more. The next step should be to demonstrate the meaning of twelve with an emphasis on the idea that twelve is composed of one group of ten and one group of two, rather than stressing the notion that twelve is one more than eleven. To help achieve this emphasis, it would be advisable first to remove the single object that was used to picture eleven as a group of ten and one more, and replace the single object with a group of two, rather than merely adding one to the existing eleven. Likewise, after writing 12 on the board and showing its connection to the subgroups it would be helpful to remove the two units before introducing 13. Thus 13 can be readily introduced as consisting of a group of ten and a group of three. A similar procedure should be followed with the other numbers from 14 to 20, with perhaps a little special emphasis on the 20 as containing 2 groups of ten and nothing more.

It is assumed, of course, that each of the preceding numbers will be developed in accordance with the pattern of going from the concrete to the abstract. Thus, the teacher will first deal with such concrete objects as books, children, blocks, tongue depressors, beads, etc. Then she can demonstrate the same number concept using pictures in books or cut-outs on the flannel board, and then before using the abstract symbols themselves, she can work with dots, squares, circles, or some other semi-abstract representations.

An analysis of our decimal money system helps to bridge the gap from the concrete to the abstract. For example, 13 cents should be thought of as one group of ten cents and one group of three cents, but it should also be thought of as one coin which stands for ten cents (the dime) and a group of three cents. The student's understanding, though, is not complete until he sees the connection between this last analysis and the technique used in our place value system of having the "1" in 13 stand for one ten just as the dime represents one group of ten.

By the time the pupil has had these experiences with numbers from 11 to 20, he should have a clear picture that the "1" in the left place indicates one ten and that the digit in the right place indicates how many ones are to be combined with the group of ten. This idea is extended and enriched through a similar study of the numbers from 21 to 99 and eventually beyond.

Here again there are many visual aids which can be utilized to sharpen and to test understandings. The conventional abacus, the open-end abacus (diagram 2), and the loop abacus (diagram 3) are among the most helpful devices.

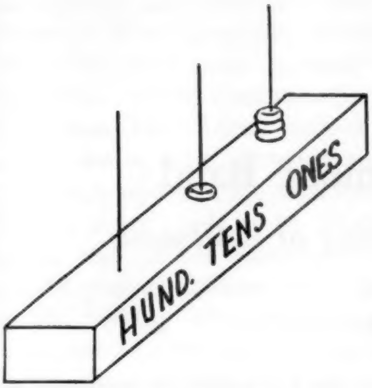

DIAGRAM 2. Open-end abacus (showing 13).

Place value cans, which are merely cans bearing one of the following labels: "Ones," "Tens," "Hundreds" are also very useful. Thirteen, for example, would be portrayed by placing one stick in the can marked "Tens" and three sticks in the can marked "Ones." Another useful device is the place pocket chart which is used in a similar manner and which is prepared in just a few minutes by pasting or stapling patch pockets on heavy cardboard. The pockets are then used just as the cans are. If the place pocket chart is made with three rows of pockets, it can be used very conveniently to demonstrate processes in addition and subtraction.

Some other useful devices include a string of 100 spools, a counting board (a board with three or four grooves cut into it in which the groove at the right stands for "ones," the second groove represents "tens," the third "hundreds," etc. It is used like an abacus. Pebbles, beads, or marbles may be used as the counters), a 100's chart, or a flannel board or magnetic board used to analyze groups or used somewhat as a place pocket chart with columns headed "Ones," "Tens," and "Hundreds."

It should be kept in mind at all times

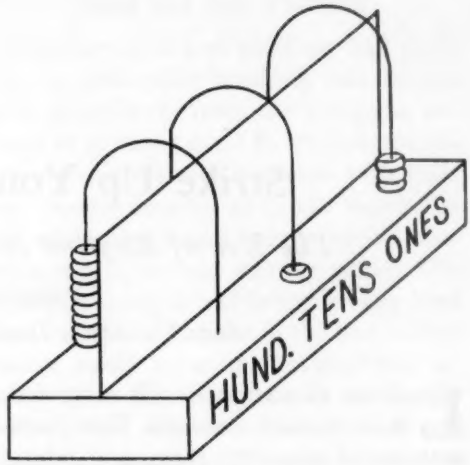

DIAGRAM 3. Loop abacus (showing 13).

that numbers take on meanings for children slowly and that only after many varied experiences is it clearly understood that our number system is a place value system in which the idea of grouping by tens plays a basic role. The varied experiences should include concrete, semi-concrete, and abstract experiences enriched by a sufficient number of teaching devices to accommodate the varying needs and abilities of the pupils.

EDITOR'S NOTE. Devices such as professor Hausdoerffer suggests are very useful in showing the make-up of numbers. His devices are easily made. Any craftsman or a good student in a shop can make each one in about an hour. Such materials should be made of sturdy materials because they will be used repeatedly. Also they should be well made and attractive so that one is not distracted by the obvious clumsiness of some of the items which teachers and students virtually "throw together."

Following the use of concrete devices is a long series of exercises with numbers as used in our decimal number system and this should be spread over several years. In upper grades, boys particularly are interested in the mechanism of simple meters such as speedometer, tally counter, etc. which show how numbers progress on our base ten. Our number system is one of the highest achievements of our civilization: it should be understood better by both children and adults.

Strike Up Your Arithmetic Band

The Use of Rhythm in the Teaching of Arithmetic

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LEARNING IN ARITHMETIC is composed of three distinct elements. First, certain patterns of quantity, form, and relationship have to be recognized, identified, and assimilated by the learner.[1] Second, a system of spoken and written symbols has to be acquired for the purpose of working with these patterns. Third, by using the symbolized patterns correctly the successful learner is able to solve problems resulting from specific concrete situations.

Let us illustrate what we mean by an example. Johnny wants to have a fleet of ten toy trucks. He already has seven. How many more does he need to have his fleet? Aspect No. 1 deals with the patterns involved, namely, the *sixness*, the *tenness*, and the "how-many-more" pattern, which is a facet of subtraction. Aspect No. 2 concerns the symbols for representing these patterns, the words and, if any writing is involved, the figures for six, ten, and "take away." Aspect No. 3 leads to the actual discovery of the solution: three toy trucks. To interpret the answer the learner must be familiar with the pattern of *threeness*.

If this analysis is accepted, arithmetic learning can be viewed as a circular process. It starts with concrete sense experiences which contain the patterns of quantity, form, and relationship. The sense experiences are by necessity concrete and specific. The patterns to be recognized in them are, on the contrary, abstract and general. In the problem-solving part of the process certain concrete and specific facts are substituted in the correctly chosen patterns, and the solution results. The circle we envisage takes the learner from many specific sense experiences through the abstract patterns back to the specifically experienced solutions of problems. We believe that the described process holds sway throughout great areas of mathematics, lower and higher. According to this view, there is nothing that divides the mathematics of the grades from the more advanced phases of the subject except that in what is traditionally called arithmetic we limit ourselves to the simplest and most commonly met patterns.

The Work of a Teacher

The first task of the arithmetic teacher is, therefore, clearly indicated. She must involve the child in a great many judiciously selected experiences from which the basic patterns of arithmetic can be derived. The difficulty of this task should not be underestimated. When, for example, the six-year-old is to get the idea of *threeness* from experience with various groupings of three things, he may very well be more interested in the non-numerical features of the experience than the purely quantitative ones. The up-to-date arithmetic teacher, eager to motivate

[1] The phrase "certain patterns of quantity, form, and relationship" is admittedly superficial. The meaning overlaps largely with the "structures and relationships" which C. Gattegno discusses in THE ARITHMETIC TEACHER, III, 3, p. 86 (April 1956).

his learning, may confront him with a picture of three dogs; but unfortunately Johnny is vastly more interested in the "dogginess" of the configuration than in its *threeness*. There are all kinds of features such as shape, color, group arrangement, etc., that compete for the child's attention. Number is not likely to be the most appealing part of the experience; and especially young children with their zest for the totality of a sense impression should not be expected to focus their attention automatically on quantity measured by number, which is, after all, the abstract product of man's intelligence seeking to survive in a number-ruled universe. Motivation in arithmetic is no easy matter. The mere exposure to concrete experiences is not enough; the children must be induced to develop a desire for dwelling on the quantitative patterns textured by the experiences. How to cultivate such a desire is a joint project for the arithmetic teacher and child psychologist.

The outlined conception of arithmetic learning is in agreement with recent trends in methodology. Experience, discovery, understanding, insight, application are the labels that tag "the new look" in arithmetic. They fit the parts of the circle described above. Out of the child's experience grows the discovery of the underlying pattern; the realization of the pattern means understanding; the various patterns seen in their mutual relationships are the basis of insight; and the final harvest comes with the correct application to a problem-solving situation. From concrete through abstract and back to concrete, from specifics through generals to the specific, that is the way of closure. And the tracks on which thought moves are laid by the symbols, both the auditory ones made through speech and the visual ones traced in writing. Without the symbolic function of the human mind—so much emphasized in Twentieth-Century philosophy—no mathematics would be possible.

Good and Bad Teaching

What are good and what are bad practices in arithmetic teaching can be decided deductively from the foregoing analysis. It is surely good to let children experience quantitative patterns with their eyes, hands, muscles as in the manipulation of concrete and semi-concrete materials and in certain number games. Obversely, it surely is bad to reduce any kind of mathematics (at least in the first twelve grades) solely to a symbol-shuffling activity which fills the student's mind with symbols and displaces from it the real things they stand for. To give a deterrent example, we may mention "rote counting" when taught as the first step in arithmetic, whereas the properly taught child evolves the sequential number names, which are symbols in sound, very gradually out of multitudinous experiences with seen and handled groupings. Never let the symbol precede the experience! Even after the symbol has been properly established, it should always be refreshed by new experience. On the basis of the above conceptions, it can be confidently asserted that semi-concrete manipulative aids are a valuable help in classroom teaching, although the experimental proof of the fact so far rests on slender evidence and needs further confirmation.[2] It can be claimed with equal assurance that the arithmetic class in which problems growing out of real life situations are neglected because most the time is devoted to computational drill is a bad class.

Rhythm in Arithmetic Learning

All of this seems an unduly long introduction to the main theme of the article, which is the use of rhythm in the teaching

[2] Neureiter, P. R. and Troisi, N. "Ol' Man 'Rithmetic," *New York State Education*, XXXIX, 8, p. 601 (May 1952). Brueckner, L. J. and Grossnickle, F. E. *Making Arithmetic Meaningful*, p. 333. The John Winston Co., 1953.

of arithmetic. The purpose was to establish a solid theoretical basis for the practical suggestions that are to follow. As we examine current practices in arithmetic, we find that the concrete experiences in vogue today are mostly of a visual nature. They require the viewing and possibly manipulating of blocks, dots, toy money, tokens, and the like. The sense of hearing is not utilized as a conveyor of direct number experiences and plays its role only after the symbolic state has been reached. Yet, thinking of music with its strong mathematical elements, one might expect that the acoustic sense, possibly combined with the sense of kinesthesia or muscle sense, would be a very willing and creative reactor to numerical stimulation. That it is so, and that the children would enjoy its being so, we shall try to demonstrate in definite procedures recommended for grades 1 through 5.

Whether we organize an arithmetical rhythm band or not, the simplest way to reach the ear with number experiences is by some kind of percussion. Outlining a rhythmic program for the elementary schools from the angle of music, Grace Fielder says: "According to some authorities, percussion is preferred because it is pure rhythm, uncomplicated by melody and harmony. . . . Children enjoy experimenting with percussion accompaniment and they should have these experiences."[3] We can clap hands, tap fingers, knock knuckles, rap sticks, strike keys, jingle bells, tingle triangles, rattle castanets, shake tambourines, clash cymbals, sound gongs, beat drums, gourds or tom-toms. If we want to go outside the field of percussion, we can whistle, toot, or honk. Any succession of clearly distinguishable sounds that serves to tally numbers will do so long as it does not disturb the peace of the school community.

In kindergarten and first grade, both cardinal (one, two, three, etc.) and ordinal (first, second, etc.) numbers can be studied acoustically. Every cardinal number word has its corresponding sound picture, and this one in turn can be related to a written numeral. As we go beyond the smallest numbers, the need for rhythm and accent will arise; and this means that the first steps toward addition and multiplication are being taken. Accents help to identify groups of strokes; and as these groups are combined, addition is represented. When the groups are equal, they lead to the pattern of multiplication. Jazz rhythm is counting by two's, the waltz furnishes an example of counting by three's, and the drill sergeant's bark, "Cadence Count," is counting by four's. In order to get the idea of ordinals across, we assign rank numbers to the children and have them make their strokes in the correct sequence; or we let them put the accent on a certain stroke in a succession, the first stroke or the second, and so on.

Subtraction grows out of the demonstration of more or less. Have two children rap out different numbers and let the rest decide, "which is more and which is less?" and "What is the difference between the two counts?" or "how many more are needed to go from the smaller to the larger number?" The basic subtraction situation represented by the take-away idea cannot be shown with sound, however. And that makes a lesson of broader philosophical implications that Johnny Brighteye may catch if it is properly explained to him. Sounds follow each other in time, not in space as marks on paper; and, being irreversible since time flows only in one direction, there is no taking away of sounds that have been produced. An important moral lesson! What has been done cannot be undone. We have to face the responsibilities for our actions.

Multiplication really becomes a study in rhythms. The group to be added repeatedly is like the measure in a piece of music, and you may place the down-beat

[3] Fielder, Grace. *The Rhythmic Program for Elementary Schools*, Mosby Co. 1952. Pp. 67.

on the first or last number of the group. The measure corresponds to the multiplicand, and the number of times it is taken makes the multiplier. Unfortunately, because factors are interchangeable by the commutative law of multiplication, we have lost a sense for the nice distinction between multiplicand and multiplier. Yet it is essential to a full understanding of the multiplication pattern. The multiplicand is the concrete grouping, directly perceived by the senses, whereas the multiplier is abstract, always involving a mental operation. With sound signals it is possible to handle the multipliers 1 and 0 meaningfully and do it more convincingly than with visual groups. 1 times 4 is one measure of four beats, and 0 times 4 is no measure at all; so the results should not be in doubt. But when you try to show the same thing with something visual like 0 times 4 marbles, some people will always argue that the marbles are there in plain sight and the result, therefore, cannot be zero.

The demonstration of division, being an inverse operation, runs into the same difficulty as take-away subtraction because of the irreversibility of anything happening in the dimension of time rather than space. We cannot start with the product, which has become the dividend, and work backward to the multiplicand or multiplier, which is now called quotient. A multiplication cannot be undone after it has been entrusted to the flux of time. It is true, the measurement concept of division can be illustrated by a question such as "how many measures of three beats are there in twelve beats?" But the actual sound experience is no different from the one used for multiplication. However, the partition concept of division is demonstrable, not in the form of "divided by a number," but in the form of "a fraction of a number." Have one child strike four times or four measures, and ask the second to strike half as much. The ratio concept of division, too, lends itself perfectly to acoustical representation. We simply strike two different numbers, one being a multiple of the other. Whereas in subtraction we ask, "how much more," we inquire here, "how many times more?" The parallel between the comparison aspects of subtraction and division is close. When we wish to demonstrate the difference between an even and uneven division, we can do it very neatly. An even division is the sound pattern that ends at the end of the measure whereas an uneven division puts in some extra beat or beats. In fact, if the class does arithmetic in a rhythm band, there will be laughter-provoking cases of some children not quitting on time and thereby failing to come out evenly with the others.

Accent the Base Ten

The broad pattern encompassing our decimal system can be concretized in sound. While with manipulative aids we bundle units into tens, tens into hundreds and so forth, we can distinguish the orders of the number system by an appropriate gradation of sound. Let one boy make the "big noise," say with a drum or cymbals, and let one stroke of it signal a bundle of ten. Then we will hear the steady, more delicate beat of the units, and every time a ten is reached, there goes the big bang from the tens' place. We can have an acoustic symbol for every two-digit number; and when the hundred is reached, let there be a sonorous gong announcing the completion of the first century mark. And so we may go on as high as we wish. By this method an appreciation may be created for the real sizes of large numbers in the thousands and millions. The youngsters will be amazed to learn that in order to rap out one billion at the rate of one number per second a person would have to spend over 31 years in uninterrupted tapping, so huge is the billion; and even a million would require over 11 days.

One of the most productive devices in present-day arithmetic teaching is the use

of the decades as support points for mental computation. For example, in higher-decade addition, such as 37 plus 8, one may think, first, of completing the 40-decade by taking 3 of the 8 and, then, of adding 5 to 40 to obtain 45. It is of great practical value in more than one way to know instantly how much any number lacks to the next higher decade. With sound it can be easily shown by designating two different types of signals for the original number and the second one it takes to complete the decade. For instance, starting with 7 we need 3 to make a ten. Let the 7 be rapped out with sticks on the desk and the 3 be added with hand claps to complete one ten. With proper modifications this can be carried into the higher decades. The decades should also be used as pegs over which to hang the multiples of the times tables. As the multiples are struck out rhythmically, now and then the big bang of a decade will coincide with a stroke and signalize the completion of a decade. Only in the multiplication of the two's and five's will there be coincidences between the completed multiple and a decade. All the other multiples (below 10 times) come out uneven with the decades.

Psychological Implications

To conclude the subject, we shall look into its psychological implications. Arithmetic learning through rhythmically measured sound opens up a new area of mathematical experience for the children. This means a powerful reenforcement of the other experiences provided in the mathematics classroom. Not only the sense of hearing is involved, but what is even more important, the kinesthetic sense. Through rhythm, numbers enter into the *musculature*. Here is a type of experience that is often deeper than one furnished by the eye. "Rhythm is an attribute of man's nature and the foundation of all art," writes Ann Driver.[4] Greek philosphers defined rhythm as order of movement. It is a feature of human nature common to all races. Primitive music is almost wholly rhythm. The popularity of the dance attests the love of rhythm in the most advanced stage of civilization. There is another peculiar feature in this conveying of number through sound. It is a concrete experience, with all the pleasure and motivation that the concrete always has over the semi-concrete and abstract, and yet it can have the quality of symbols. For instance, in the Morse code the transmitted sounds are symbols for letters. The concrete, semi-concrete and symbolic stages of arithmetic learning coincide.

As we broaden the base of experience from which quantitative patterns are derived, we are promoting another objective of mathematics instruction, namely, the recognition that the abstract patterns we are studying are truly general. To be able to identify a certain pattern in all kinds of different guises is the mark of the good mathematician; and when we shift from the eye to the ear as our conveyor of mathematical experience, we are certainly becoming aware of the pervasive character of our general patterns.

Finally a word about motivation. We can safely assert that anything resembling an arithmetical rhythm band is bound to increase the popularity of early learning in mathematics. The never-ending chatter and blare of light music from radio and television sets testify to the love of rhythm among the masses. That rhythmic exertion is enjoyed is proved on every dance floor. And that children, after a period of quiet desk work, will love to let loose with noise-making, muscle-exercising activities is well-known to every teacher. So here is a pleasurable road to learning, one that serves as a channel for the energies of youthful bodies.

[4] Driver, Ann. *Music and Movement.* Oxford University Press, 1938.

There is a strong element of team work in it. Rhythm is the natural device for producing the team spirit, as is illustrated in a band or a marching unit. Grace Fielder lists as sociological objectives of rhythmic activities:[5] "1. Development of group consciousness. 2. Development of the desire to contribute to group activities. 3. Development of the ability to conform to group standards." By having the willing followership of the children the teacher will strengthen her own position as a leader. But she can also delegate leadership roles to members of the class. Here is an opportunity for real group learning and a measure of elementary group dynamics.

The modern arithmetic class needs group leadership. We have seen one of the most useful aspects of elementary mathematics instruction go by the board in the last two decades, namely, mental arithmetic. One reason for its disappearance was the dissolving effect of a theory of classroom management that stressed individual pupil work and practically abolished the teacher's role as a group leader. Another reason is the sad fact that many teachers are not capable of holding the attention of a whole class for any considerable length of time and for a sustained mental effort. Through rhythmic group activities, developed from the primary grades on up, we may help to restore classroom control and establish an attitude favorable to the later cultivation of mental arithmetic. Recognizing mathematical patterns by the ear demands a firmer hold of the attention than the mere seeing of visual patterns. Because sound is invisible, there is a feature of abstractness about it that requires the exercise of both attention and imagination. Without these two qualities, proficiency in mathematics is impossible. We claim, therefore, a special value for acoustic and rhythymic arithmetic within the general frame work of mathematics instruction. STRIKE UP YOUR ARITHMETIC BAND!

[5] *Loc. Cit.*, p. 44.

EDITOR'S NOTE. Many people realize the close relationship between mathematics and music. Professor Neurieter has explained how this may be capitalized in the elementary school. Some teachers are a little afraid of music because of their lack of formal training therein. But all of us can understand simple rhythms. We should not be afraid to experiment and explore in our modes of teaching: we may even become discoverers. We know that some children respond more readily to sound in cadence than others just as certain ones progress more rapidly to abstractions that are purely mental. In our teaching we should make use of multi-sensory aids to learning. Let us not lose ourselves so much in the "music" of the arithmetic band that we fail to discern and learn the arithmetic!

Teaching Quantitative Relationships in the Social Studies

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THE SOCIAL STUDIES UNIT has become recognized as the part of the elementary school instructional program which should serve to synthesize much of the learning which occurs during the remainder of the school day. While most schools set aside time when a major instructional effort is directed toward helping children develop proficiency in and understanding of such learnings and skills as are involved in reading, writing, arithmetic, spelling and language, it is in the social studies unit that these learnings and skills are applied in functional situations. It is here children read to gain information, prepare and present oral and written reports, plan and participate in dramatic and music activities related to the unit, and engage in many construction and processing projects. All of these activities contribute directly to the attainment of social studies goals and concurrently present the child with situations in which he may put to use reading, writing, reporting, planning and other knowledges, understandings and skills. It is in this way, too, that the child will encounter countless opportunities for practical experiences with quantitative concepts. In short, there should exist a two-way and mutually supportive relationship between the social studies unit and other areas of the school curriculum. From the standpoint of sound instructional procedure, this two-way relationship serves at least three important purposes:

1. It supports, strengthens and reinforces the related learning by giving additional practice in its use in a functional setting.
2. It builds an awareness, readiness or need for the related learning by calling attention to its application and use in daily living. This may, in some cases, serve as a point of departure for basic instruction in the related learning.
3. It makes social studies material more comprehensible for the young child by clarifying concepts from related and allied areas of learning which are basic to the social studies topic under study.

The integration of various facets of the language arts, music and art in social studies units has established itself as standard practice in most classrooms, but there has been a tendency to overlook possibilities for a similar integration of quantitative relationships. Perhaps teachers are not aware of the quantitative aspects of social relations and the social studies. Much of the content of the social studies deals with properties which are largely qualitative in nature—values, judgments, attitudes, feelings, and various subtleties of human behavior. The problem of quantifying qualitative data has obstructed the exact and rigorous research in human relations which has characterized research in the physical sciences. But authors in the field of arithmetic have long called attention to the social phase of arithmetic in addition to the computational phase. The fact that arithmetic clearly has a social phase means there must be many cases in which mathematical concepts are used in the social intercourse of people. The social studies, which deal with social relationships, must, therefore, logically present many occasions for the teaching and use of such concepts. The subsequent discussion points to some of the quantitative aspects of human relationships and suggests ways in which a substantial contribution can be made to the child's understanding of quantitative concepts within the context of the social studies.

Developing Sensitivity to the Use of Mathematics in Human Relationships

As modern nations of the world have advanced into the scientific and technological age, there has been correspondingly greater use of mathematics in human relationships. The allusion to or direct use of numbers in one form or another has a way of manifesting itself unexpectedly in any discussion of human affairs. This is evident at the earliest levels of the primary grades when children and their teacher discuss the problems of living:

> "Who can go to the calendar and show us what day this is?"
>
> "How many brought money for the school savings today?"
>
> "I need five boys to help me move some books."
>
> "We will use only half a piece of paper."

This is equally evident when an adult picks up his evening paper and reads of the human affairs of the day at the national and international level:

> "Thousands of refugees stream across Austro-Hungarian Border."
>
> "President grants $100,000 for emergency aid."
>
> "Employees granted 8 per cent hike in salary."
>
> "Local community spends $50,000 in park development."

People are continually making references to how many, how far, how much, who was first, how long ago, how far into the future, etc. When a society becomes highly complex and industrialized, quantitative relationships become an indispensable part of the daily lives of everyone. The entire social, scientific and technological milieu of the contemporary world is mathematical in character.

A close examination of social studies reading material prepared for children will also reveal the extent to which mathematical concepts are present in human relationships. Phrases such as the following pervade a considerable amount of material written for the middle grades: "for two hundred years," "it became a much larger city," "the amount of cotton," "about 25 to 30 acres," "hundreds of different things," "a few feet under the surface," "up river for 100 miles," "about twice what it is now," "rainfall about 11 or 12 inches," "leading crops," "three day's march," "eighty miles away," "increased greatly in the first hundred years." The following paragraph illustrates the heavy loading of number concepts which may be present in a single paragraph of social studies material:

> "Listen to these figures: There are more than 400 piers and wharves. There are more than 700 miles of space for loading and unloading ships. About one ship enters or leaves the harbor every hour, day and night. Some 250,000 workers earn their living in the shipping industry. Almost half of the goods imported and exported by the United States pass through this harbor. New York Harbor is America's most important gateway."[1]

The constant reference to quantity in the discussion of human affairs has a conditioning effect upon children and adults which dulls one's awareness of its presence. *Mathematical concepts are bandied about in social studies instruction as if they were easily understood by the learner. In many cases they are not understood at all; in fact, the learner may hardly be conscious of their presence.* Teachers should take time out frequently to call attention to and explain the quantitative aspects of social studies topics being studied. Such teaching procedures invariably help the child gain a better understanding of the social studies as well as the mathematical concepts related to the topic at hand.

Giving Children Experiences in Reading Mathematical Material

The extent to which quantitative concepts are found in social studies, as previously noted, makes it possible for the teacher to give numerous experiences in

[1] O. S. Hamer, D. W. Follett, B. F. Ahlschwede and H. H. Gross, *Exploring Our Country*. Chicago: Follett Publishing Company, 1953, p. 153.

reading mathematical material. These experiences may take the form of simply reading numbers; reading and interpreting mathematical data found in tables, charts, graphs and maps; being able to make comparisons of significance between quantities; being able to make inferences from mathematical data; and developing an understanding of the vocabulary of quantitative data. Reading experiences of the type just described acquaint the child with the need for accuracy and exactness when one deals with certain types of social data and constitute an important part of the child's education.

Social studies materials frequently include many indefinite references to time and quantity which also need to be singled out for special attention by the teacher. For example, in learning about the growing of cotton, the child may read that cotton " . . . needs much sunshine and much water. It takes much heat, light and moisture to make the plants keep on blooming." How much is *much* in each of the cases where the term was used in the preceding sentence? What time interval is intended, for example, in the following sentence? "For a long time most of the people of New England were farmers." Five years seems like a "long time" to a fifth grader, but obviously the people of New England were farmers for a more extended period than five years. A half-hour might be considered a "long time" if one were near death and was awaiting the arrival of a doctor. Indefinite references to time, space and object-quantities may be very confusing to the young reader and call for clarification and teaching. This can best be done when such terms are encountered in their natural habitat—the social studies.

Developing an Understanding of Measurement

Measurement of space, time, and object quantities occurs extensively in the consideration of social studies topics which the child needs to be able to handle skillfully. Commonly the measurement of space, time, and object quantities are related to one another in some way. Travel is measured in distance traveled in a designated amount of time. Rainfall is measured in inches for a season or year. Product output of a region is measured on the basis of a specified amount of time. Moreover, the relationship of all three types of measurements may be shown in a single representation such as the line graph. In the line graph the child must deal with changes in quantity over measured amounts of time, represented by equally spaced lines on a grid-work. Other than arithmetic itself, perhaps no other area of the elementary school curriculum deals as continually with measurements of space, time, and object quantities than does the social studies. Even a cursory glance at a social studies textbook for children in the middle grades will demonstrate the numerous references made to the measurement of distance, to time intervals between date-events, and to quantities in the form of production, amount of rainfall, density of population and others. A visit to an elementary classroom where children are involved in a social studies unit will find them measuring paper for a mural, following a recipe for making cookies, making time lines, measuring distances on maps, constructing graphs and many similar activities involving the use of measurement.

While many examples could be drawn to show how the imaginative teacher helps the child develop his understanding of measurement within the social studies unit, the following illustration will demonstrate sufficiently well how measurement is involved in but one aspect of the unit. Social studies instruction always makes some use of maps. Maps are representations of the earth's surface which graphically reduce the area represented. Intelligent map use necessitates a facility with such sub-skills as reading the scale, converting scale into miles, developing a

sense of ground and map distance, comparing relative size of various land areas and water bodies and understanding various methods of expressing scale. All of these sub-skills demand the use of measurement in one form or another. Furthermore, map study provides an excellent setting in which to teach one of the basic concepts underlying the measurement of space—the need for a point of origin and related points of reference if one is to locate oneself in space with any degree of exactitude. The teaching of concepts of measurement in connection with map study is, therefore, no academic frill but an absolute prerequisite to the understanding and use of map skills themselves.

Providing Practical Computational Situations

The teaching of fractions and fractional parts is regarded as a fairly difficult process and, therefore, is ordinarily deferred until about fifth grade. Yet research has repeatedly indicated that many children in first grade have a functional knowledge of certain simple fractional relationships. This apparent contradiction serves to demonstrate that rather complex computational procedures can be performed by children providing the situation is within their realm of experience and is taught at a level of abstraction commensurate with their degree of maturity. Problems encountered in the course of the social studies unit oftentimes satisfy these two requirements and present the child with practical situations in which to use computation. The practice of writing artificial "story problems" is unnecessary if the teacher is alert to the many real problems which arise daily in the social living of the classroom—many within the social studies. A few examples will show this to be true:

A first grade decides how many cars will be needed to take the group on a field trip to the dairy.

A fifth grade wants to compare the cost per acre of the Louisiana Purchase with the price of land locally at present-day prices.

A fourth grade needs to find out how much money to collect from each member of the class to pay for the expenses of a class party.

A third grade computes the cost of materials to build a cage for its pet.

A sixth grade plans its expenses for a week's camping experience.

A fourth grade keeps the records of an animal experiment in a social studies unit on nutrition.

The examples given are not exceptional nor unique. Situations such as these arise in every classroom.

Elementary teachers frequently encounter the disturbing circumstance of having taught children the fundamental processes well and yet find them unable to apply those learnings to problem-solving situations. The children seem to have mastered the manipulative and mechanical aspects of computation but fail completely to relate such learnings to the solution of everyday computational problems. This non-functional knowledge of arithmetic is an indication that the social phase of arithmetic is being neglected in the program of arithmetic instruction and needs to be strengthened. The use of number operations in the solution of problems which arise daily in social studies activities will aid in this respect.

Children need many problem-solving experiences of this type, for they represent the kind of computational situations met most frequently in the ordinary course of daily living by the average citizen. He wants to compute problems dealing with the family budget, how he can get the best buy, how many miles per gallon of gasoline his car is currently yielding, what expenses he will incur in a family vacation, how to determine the caloric content of his diet, and countless others. Early beginnings are important in developing skill in the application of number operations to the problems of living.

Summary

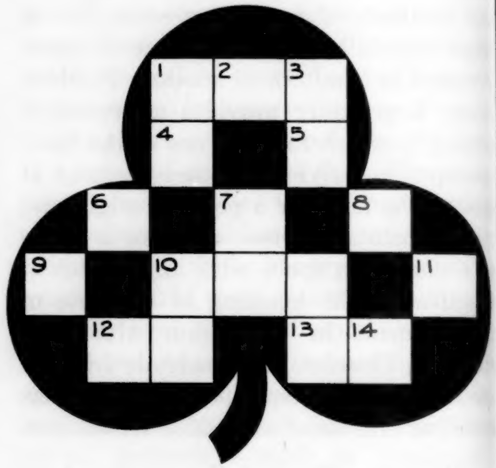
The foregoing material has summarized briefly some of the quantitative aspects of

the social studies and has suggested ways in which quantitative relationships can and should be taught within the framework of the social studies. While the social studies cannot be expected to carry the entire responsibility for helping children develop arithmetic competencies, it does provide many situations which will help children (1) build sensitivity to the use of mathematics in daily living, (2) learn to read mathematical material as related to human relationships, (3) develop an understanding of measurement, and (4) gain practice and skill in certain aspects of computation in the solution of practical problems. Furthermore, the clarification of mathematical concepts is indispensable to the understanding of social studies material under consideration. The most important factor is an alert and imaginative teacher who is himself aware of the quantitative nature of human affairs and is sufficiently concerned about its importance to seize upon the many possibilities for teaching quantitative relationships within the social studies.

EDITOR'S NOTE. Yes, for most people, arithmetic should be a handmaiden who is ever present and helpful in sensing, in interpreting and understanding, and in carrying to a valid conclusion the essential mathematical concepts and relationships in our society. One study showed that 70% of divorces originated in failure to grasp and contend realistically with economic data. We do live in an industrial era with numbers and quantities constantly necessary to record events. How can we be intelligent if we do not understand these things? Why do so many adults skip the numbers they encounter in newspapers and magazines? Who will pay the piper?

Certainly a good teacher will provide a basis for understanding quantity whether this be in music, arithmetic, or social studies. Also, a good book will provide information that is correct and is presented in well written statements. Recently the editor met a school superintendent who laughed about a problem he had written (as a student teacher) many years ago and in which the answer resulted in a baby weighing 120 pounds. His supervisor merely asked if he was thinking of a baby horse. The lesson, he said, had served him well for 25 years.

A Cross Number Puzzle for St. Patrick's Day



ACROSS

1. Product of $3 \times 9 \times 7$.
4. The unit's digit in the date of St. Patrick's Day.
5. The ten's digit in the date of St. Patrick's Day.
6. March is the ______ month of the year.
7. The number of prime numbers between 1 and 20.
8. The least common denominator of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{5}{8}$.
9. The altitude of a triangle whose base is 12 units and whose area is 42 square units.
10. The number of days from St. Patrick's Day to Christmas. (Do not count St. Patrick's Day or Christmas Day.)
11. The number of letters in the name of the color associated with St. Patrick's Day.
12. The square root of 1936.
13. The number of degrees in a right angle.

DOWN

1. The day of the month on which we celebrate St. Patrick's Day.
2. The number of sides an octagon has.
3. The average of 97, 98, 94, 80 and 86.
6. The number of sides of a triangle.
7. Two angles of a triangle are 42° and 50°. How many degrees are in the third angle of the triangle?
10. Two dozen.
11. A pentagon has ______ sides.

Contributed by MARGARET WILLERDING of San Diego State College, California

Johnny Can Learn Arithmetic

The Report of an Arithmetic Contest

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ON SATURDAY MORNING, May 12, 1956, three of the main meeting rooms of the Milwaukee Engineers' Society Building were extraordinarily quiet, extraordinarily so, because around the tables sat first, second, third, and fourth grade children listening intently.

Before each child was a pencil, a printed test form, and a Numberaid Abacus. The final words of instructions from the educators who served as judges were given. Stop watches in the hands of the Vocational Guidance Committee members of the Milwaukee Engineers' Society were checked, and a new kind of competition was under way. Forty minutes later, the preliminary round of the first Abacus Contest ever held in the United States was over.

A cherub of a boy, a first grader, one of the sixty-odd youngsters on the way downstairs for ice cream and cake, was heard to say, "That wasn't hard. If there had been more time, I'll bet I could have worked them all, without a mistake, too." Sixteen of the semi-finalists were chosen to participate in the final contest, held Saturday afternoon, May 26, at Radio City. The final contest was televised by WTMJ-TV as one of its weekly educational programs, "Let's Experiment," sponsored by the Milwaukee Museum.

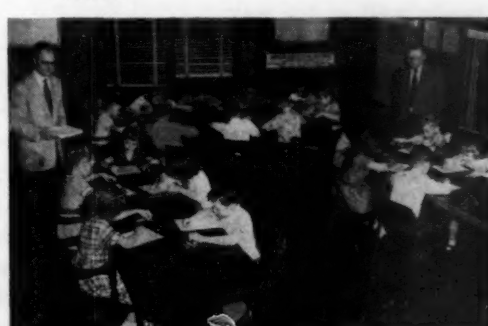
The contest was divided into five sections of four minutes each. The first section of the test contained numerical addition problems; the second, multiplication; the third, subtraction; the fourth, division; and the fifth, word problems in each process and in combinations of processes.

The four first-grade finalists correctly completed an average of:

1. Nine problems in addition which contained one- two- and three-digit numbers involving carrying.
2. Twelve problems in multiplication, involving multiplication of two-digit numbers by one-digit multipliers, and involving extended multiplication in any column.
3. Eleven problems in subtraction of numbers containing four digits, involving borrowing in any column.
4. Twelve problems in division, which contained three digits in the dividend and one in the divisor, with remainders.
5. Four word problems which contained addition, subtraction, multiplication and division.

The four third-grade finalists correctly completed an average of:

1. Twenty-three addition problems, extending to three-digit addition of seven addends, which contained two decimals.
2. Twenty-five multiplication problems, extending to four-digit multiplicands and three-digit multipliers, containing decimals in both multipliers and multiplicands.



Second grade students from the Milwaukee, Wisconsin Metropolitan Area, under the supervision of Mr. Jack Young, Principal of Jefferson Public School, use the Numberaid Abacus in the first Numberaid Abacus Contest sponsored by the Milwaukee Engineers' Society.

3. Twenty-four subtraction problems, extending to five-digit minuends and subtrahends, containing decimals in both minuend and subtrahend.
4. Twenty-three division problems, extending to division of a three-digit dividend by a two-digit divisor, with remainders.
5. Eleven word problems involving all four processes, as well as measurement and determination of an average.

The purpose of the contest was to give students in the primary grades the opportunity to demonstrate their achievement as well as the potential of the Numberaid Abacus as a tool for teaching arithmetic. Many of our schools do not permit children to study arithmetic formally in Grades One, Two, and Three, thereby handicapping them in the development of basic arithmetic at an age when such study is a source of pleasure to them.

The climax of the competition came the following Saturday afternoon when prizes and certificates were presented to the finalists by the President of the Milwaukee Engineers' Society, Mr. Frank Roberts.

The success of the first competition has resulted in setting the dates for the second Numberaid Abacus Contest. The preliminary competition is to be held Saturday morning, May 4, 1957. The final competition is scheduled for Saturday afternoon, May 18. Planning for the preliminary round of the second Numberaid Abacus Contest to be held May 4 is already well under way.

More than twenty schools in the metropolitan area of Milwaukee will participate. The Numberaid Abacus is being used extensively in Norfolk, Virginia; Sacramento and North Sacramento, California; New Castle, Delaware; Culver, Indiana; West Hartford, Connecticut; and Birmingham, Michigan. These areas are watching developments with interest, and may even attempt to organize their own contests this year.

EDITOR'S NOTE. Dr. Schott has reported rather amazing achievement in computation by these pupils in grades one and three through the use of the Numberaid Abacus. Teachers who would like a copy of the tests used and more information about the Numberaid Abacus may write to Dr. Schott at 3566 Frederick Ave., Milwaukee.

One asks whether these same children are learning to compute in the conventional way as well as with the aid of an abacus and if they are developing an understanding of arithmetic and its uses in our society? Should we return to modes of computation that have been neglected for several centuries? What are our goals in arithmetic instruction and how are these best achieved? Do we need very different kinds of schools and textbooks and learning materials? In our highly industrial society in which every intelligent person needs a great deal of sensitivity to things arithmetical and an understanding of many concepts and the ability to think and to compute in order to draw a tenable conclusion, arithmetic is very important. How can our young people best achieve these competencies?

New York State Arithmetic Conferences

The State Education Department of New York is holding a series of regional conferences on arithmetic for school superintendents, principals, and supervisors. The aim is to give administrative and supervisory staff an opportunity to learn what constitutes a modern program in arithmetic and to gain a good deal of understanding of content and method. Each conference features addresses by well known leaders in the field plus six hours of group work on topics such as: numbers and number system; fundamental processes; fractions, decimals, and per cents; and problem solving. This sounds like an excellent idea.

The Day Camp and Arithmetic

B. J. Goodrich
Miami, Florida

The Day Camp project of a fifth grade class at Little River School which was held at Greynolds Park, Miami, Florida, on April 17, 1956, afforded many pleasant educational experiences.

In addition to the regular courses, Plant Life, Rocks and Soils, Food Preparation, Camp Safety, and Creative Activities, special emphasis was placed on the arithmetic involved in a successful camp.

The number in attendance including pupils, leaders, mothers, teacher, and bus driver, was forty. The committees selected to direct the program were: *survey*, *purchase*, *materials*, and *clean-up*. The time consumed for the whole project was three weeks. Two weeks were used for preparation and one week for follow-up.

The following is a summary of arithmetic concepts and methods involved in the entire camp project.

1. Adding the number of pupils, leaders, and guests to get the total.
2. Adding to determine the total cost of food.
3. Multiplying to find the cost of transportation at 25¢ per child.
4. Multiplying the number of pupils by 25¢ (the usual cost of lunch) to find the amount of money available for food.
5. Subtracting the value of any donated food from the total cost of food.
6. Dividing the rent of the camp ($6.50) by the number of students to determine the individual cost.
7. Adding the cost of bus, food, and rent, to arrive at the total camp cost. Then dividing by the number of pupils to get the total individual cost.
8. Buying by dozens, pounds, etc., measuring, weighing, taking note of fractional parts, and making change.
9. Using the calendar to determine the amount of time for preparatory study before camp date; allowing five days for follow-up.
10. Having the survey committee check the speedometer to determine distance to camp, and at the same time check by the watch to learn the amount of time to be consumed in travel, in order to know the amount of time available for the camp program, and still return to school in time for dismissal at 3:00 P.M.
11. Planning the schedule for classes and the noon meal according to this allotted time; allowing ample time for clean-up.

The menu selected by the class was wieners with sliced onions and mustard, baked beans, cookies, marshmallows, Kool-aid, and buns.

A study was made of the food store newspaper ads for several days to find where the food could be purchased at the lowest cost.

Arithmetic Problems

We were faced with many types of problems, and in order to have a successful educational camp, these problems had to be solved.

We went to work on the following problems:

1. How many wieners will it take for 40 people, if each person eats two?
2. At 10 wieners to a package, how many packages will it take?
3. What will be the total cost of the wieners at 45¢ a package?
4. What part of a dozen is one package of wieners?
5. There are 8 buns to a box. What part of a dozen is this?

6. How many boxes of buns will we need? How many dozen buns will this be?

7. At 4 ounces per person, how many ounces of baked beans will it take? How many pounds will this be?

8. A No. 10 can of beans holds 7 pounds and 2 ounces, and costs 71¢. A No. 2 can holds 12 ounces and costs 10¢. How much will it cost to feed 40 people using the No. 10 can? How much will it cost using the No. 2 can?

9. Which is cheaper, the small cans or the large? How much cheaper?

10. If one package of Kool-aid makes 2 quarts of drink, how many glasses would that be?

11. Allowing 2 glasses per person, how many packages of Kool-aid will it take for 40 people?

12. What would be the cost of the Kool-aid at 5¢ a package?

13. What fractional part of a gallon would one package of Kool-aid make?

14. How many gallons will it take to serve 40 people?

15. If the pitcher holds $\frac{1}{2}$ gallon, how many times can it be filled from 5 gallons of Kool-aid?

16. Estimate the amount of Kool-aid, if there is 5 gallons of water, the equivalent of two gallons in ice cubes, and $\frac{1}{2}$ gallon in sugar?

17. If it takes $\frac{1}{2}$ cup of sugar for each quart of Kool-aid, how many cups of sugar will it take? How many pounds will that be?

18. If sugar is 9¢ a pound or 5 pounds for 39¢, how much money will be saved by buying the 5 pound bag?

19. How much more will we have to spend, if we buy a jar of mustard at 19¢, 2 bags of marshmallows at 25¢ a bag, 1 box of paper napkins at 13¢, and 2 bags of ice cubes at 40¢ a bag?

20. If the 10 boxes of wiener buns that are being donated, would cost us 21¢ a box, how much would this be?

21. If the cookies which are being donated would cost 19¢ a dozen, how much would 40 cost?

22. How much will be saved by having the cookies and buns being donated?

23. Add the total cost of food.

24. Add the cost of food to the cost of bus, and the cost of camp rent. What is the total cost?

25. Divide the total cost by the number of pupils. What is the cost per pupil?

26. If the bus leaves school at 8:30 A.M., returns by 3:00 P.M., and uses $\frac{1}{2}$ hour in travel time each way, how much time can be spent at camp?

27. If one hour is used for lunch and clean-up, $\frac{1}{4}$ hour for devotions, and $\frac{1}{4}$ hour for final clean-up, how much time will be left for classes?

28. How many $\frac{1}{2}$ hour classes can be conducted in this time?

At the conclusion of camp, it was the unanimous agreement of pupils, parents, and instructors that the day rated extremely high in enjoyment and educational value. There was no doubt about a wider grasp, and a new understanding, of arithmetic concepts and values.

EDITOR'S NOTE. A camp situation can be very fruitful in developing concepts, procedures, processes, and problems. It is especially valuable to pupils when they take an active part in the planning-thinking stages as was done by this fifth grade at the Little River Elementary School. But arithmetic learning must not stop with the camp. New items of information and concepts and processes which were developed need additional practice so that they become well established. In many activity situations the whetted interest of pupils serves as a starting point for renewed learning. It is difficult to gauge just how much a factor in learning such things as attitude and atmosphere serve.

Reading in Mathematics

GEORGE W. STREBY
Anacapa Junior High School, Ventura, Calif.

SOMETIMES THE TEACHER thinks that the students cannot work verbal problems because they do not have the ability to read. This is a vague statement because the act of reading is very complex, involving many skills and understandings. What does the teacher mean when he says the student cannot read? Does this mean that the student cannot pronounce well, does not read smoothly, does not understand the concepts, or does not understand the technical words used in the problem? Stating that a student cannot read is making use of the catch-all word that may mean any number of things.

Reading verbal problems in mathematics texts does require a different technique than reading descriptive material or fiction. The inability to grasp the full meaning of a statement is more serious in mathematics than in any other subject. Teachers of mathematics must insist that students understand fully verbal expressions of the subject.

Studies have been made of the reasons why some pupils have so much difficulty in solving problems. These studies reveal that the reading of arithmetic problems is different from reading done in other subjects. It is the purpose of this paper to point out some of the most outstanding difficulties that are due in whole or in part to lack of reading ability and to tell in a general way some of the things that can be done to overcome these difficulties. The data for the improvement of these difficulties was gathered from studies that have been made on this particular subject, experience and observation as a mathematics teacher.

Problems in Reading Mathematical Material

Some of the difficulties that pupils encounter in their reading of mathematics are caused by:

1. Unawareness of the nature and purpose of reading in mathematics.
2. Insufficient technical vocabulary peculiar to the field of mathematics.
3. Inability to distinguish between relevant and irrelevant material.
4. Inability to relate pictorial and tabular material to verbal material.
5. Inability to recognize and get the significance of key words.
6. Failure to use all the steps in the solution of the problem.
7. Inability to relate previous materials to the present reading problem.
8. Inability to use the mathematics text as an aid to study.

Improvement of the Difficulties

I. *Making pupils aware of the nature and purpose for which they are reading.*

One of the first things a teacher can do toward eliminating reading difficulty in mathematics is to make the pupils aware of the nature and purpose for which they are reading. Reading in mathematics is work-type reading. It must be slow and intensive. To read mathematics efficiently, one must be able to note details and evaluate them, to follow directions, and to organize facts and relate them one to another. If pupils have not learned to do this kind of reading they must be taught to do so. In other subjects pupils may be using techniques for extensive reading but they will find that in mathematics these techniques are insufficient for comprehension.

The teacher should have pupils read problems aloud to see if they are reading

them correctly. If there has been an error in reading, there should be a discussion as to whether or not the error would affect the mathematical solution.

Pupils should know that they are not ready to start solving a problem until they are completely familiar with the details. To be familiar with a problem involves building reading the problem more than once. The problem should be read first rather quickly to get the general thought; second, reread the problem to get all the facts needed for the solution; and finally, reread again some parts of it to check to see that the solution is complete.

II. *Building vocabulary*

Teachers must make pupils realize that their success in mathematics depends to a great extent on understanding the precise meanings of mathematical terms. It must be pointed out to the student that words which they are familiar with in other situations may carry a different or added concept in mathematics.

The teacher must not assume that pupils know the meaning of mathematical terms, symbols or abbreviations. All these things that might afford a stumbling block in reading the lesson assigned, should be isolated and deliberately taught. New words should be defined, illustrated and spelled. Teachers should use any means at their disposal to establish the concept of new terms before the pupil meets them in a reading assignment.

III. *Getting the significance of key words:*

Pupils fail to do their problems correctly sometimes because they do not recognize or get the significance of key words when reading. This inaccurate type of reading is caused by failure to weigh the words properly; importance is given to the wrong word in the sentence or key words are completely ignored. Skimming over the material will cause this.

Oral reading of the problems and discussion of why an "answer is wrong" will make pupils aware that careful analytical reading will help them eliminate mistakes.

IV. *Distinguishing between relevant and irrelevant facts:*

Many pupils fail to solve problems correctly when the problems contain data that are not essential to the solution. Many pupils think that they must use all of the numbers to solve the problem. To help pupils with a difficulty such as this, the teacher should give practice in reading problems that have non-essential ideas, stressing how to read to get the general impression and how to re-read to put the facts in the proper relation to each other.

V. *Including all the steps:*

Pupils omit steps in a problem because they do not understand the fundamental processes involved in its solution. However, leaving out steps might also be due to the fact that the pupils have skipped pertinent materials in their reading. The teacher should give specific drill in reading problems and placing emphasis on important parts by asking questions about the different parts of the problem. Considerable practice must be given in mastering steps in "how to solve" rather than "solving" the problem.

VI. *Relating pictorial and tabular materials to verbal material:*

Teachers must impress pupils with the fact that reading verbal materials accompanying graphs, tables, charts, etc., is only one step in their interpretation. Being able to interpret these types of materials intelligently involves making a relationship between the reading materials and the pictorial or tabular materials. Proficiency in seeing relationships will come only with practice of handling all kinds of graphic materials. Such practice must include making graphs and tables from verbal materials as well as preparing discussions for graphic materials.

VII. *Relating previous material to what is being read:*

Besides reading carefully to get the facts and practicing on vocabulary to get the understanding of new terms, pupils must do reflective reading—that is, thinking over what they have read and relating it to other things that they have learned before. While this is a study skill helpful in any situation, it is especially helpful in mathematics where sequence is so important that an understanding of earlier lessons is essential to the reading of a later one. It is important that pupils read in such a way that they understand each section clearly before trying to move on to the next.

VIII. *Using the mathematics textbook:*

Many pupils make little use of their textbooks except for the lesson at hand. They either do not know, or they do not bother to use their book as a source of clarifying and extending concepts or as an aid in reviewing.

Teachers can help pupils overcome this handicap by teaching the parts of a textbook and how each part is used. This should be done at the beginning of the school year. Definite lessons in using the parts of a textbook and insisting that pupils find the answers to questions for themselves are recommended procedures.

Conclusion

To some extent the language of mathematics undergirds the decisions and facilitates the reports that citizens in all walks of life have to make. All people need number sense as well as common sense.

We know that a person is not really reading unless he gains meaning and understanding from the words he sees. We know that mathematics plays a very important part in solving problems that every adult has. We also know that it is one of the basic needs of every pupil in school.

One of the pressing problems for many teachers in mathematics is how to help boys and girls who have trouble reading mathematical materials. Whatever technique is used to solve this problem should provide the pupil with many varied, interesting, and challenging experiences based on their problems, needs and abilities.

EDITOR'S NOTE. Mr. Streby has called to our attention the difficulties that pupils encounter in the reading of word problems. He offers advice on how to help pupils who are in difficulty. Reading in Mathematics is not the same as the reading of simple plot-interest narratives. In arithmetic we use both a general and specialized vocabulary and back of any vocabulary must be a great deal of learning if we are to work above the level of non-sense. For his own school Mr. Streby and his colleagues have developed an extensive vocabulary list of the mathematical terms which pupils should understand and be able to identify and use in their thinking. It is suggested that the preparation of such a list with approximate age-grade designations would be a worthwhile project for teachers in any school. It is a project on which pupils may work. In one school this was an "extra job" for bright pupils to keep them fruitfully occupied so they did not waste their time. Should THE ARITHMETIC TEACHER print a vocabulary of mathematical terms used in lower grades? One for intermediate grades, etc.? Who would like to prepare such lists and tell how they should be used?

Arithmetic Can Be Fun

Mrs. Norene Harmon
Patterson Elementary School, Patterson, Calif.

To some six- and seven-year olds, arithmetic is a dull and fearful subject, to others fun and frolic. This is how one class made it fun and frolic.

A Class Store

The teacher introduced the use of money by having the children count their lunch money and milk money each morning. The children studied a chart of real coins, and a toy store was suggested.

A small toy store was brought into the room. The store was made of plywood about three feet long and four feet high. It had a small door and large windows with three shelves. The pupils brought some of their inexpensive toys to be placed on the shelves. Pencils, balloons and suckers were placed on the top shelf. That was the penny shelf. Articles for a nickle were placed on the second shelf and the ten cent collection was on the third shelf.

To afford an opportunity to play store and to work with varying amounts of money, the children selected two or three articles and then told their money story.

Playing store activities were continued by having the children pretend to go to the store and buy two or three toys, illustrating their purchase on paper, and then telling how much money they had spent. Other exercises were employed, too. Each child was given twenty-five cents to spend. Children then illustrated their stories and told how much change they had left. Members of the class also suggested that three nickels might be used in place of a dime and nickel. A child had to demonstrate his ability to make change before he could become the storekeeper. The same steps in making change that are used by clerks in stores were used in this activity. They started with the amount the item cost, counted pennies to the nearest multiple of five, then counted nickels and dimes to a quarter. They decided it was fun using the toy store, changing the articles on the shelves and being store keeper with access to the toy cash register. Repetition of this experience was always welcome.

During this activity the children were studying about Community Helpers. A health unit was in progress and plans were being made to visit the hospital just before Easter. It was suggested that the class take a large Easter basket filled with appropriate toys for the children confined in the hospital. Where would we get the money to buy the gifts?

A candy sale at noontime was planned. Letters were written to mothers asking them to make candy for the sale.

A large chart of the plan was written and the children advertised the candy sale in each room of the school:

> *We will have a candy sale next Friday at noontime. The candy will cost five cents a bag. We plan to buy presents for sick children in the hospital. We hope you will help us by buying some of the candy.*

Arithmetic became real again.

"What day should we sell the candy?" (use of the calendar)

"What time will we sell the candy?" (use of the clock)

"What expenses will we have?" (use of money)

"How big will we cut the pieces?" (use of ruler)

"How many shall we put into each bag?" (measurement)

"Who will sell the candy?" (use of tally by voting)

After the candy sale, the class was delighted to learn they had $23.60. The money was carefully counted, and it was observed that 100 pennies or 20 nickels, or 10 dimes, or 4 quarters or two half-dollars made one dollar. Cash received included a one dollar bill and a check for one dollar which was later cashed at the bank after proper endorsement.

Next plans were completed for the trip to the hospital. Cellophane bags used in sacking the candy, a large potato basket to be decorated as the Easter basket, crepe paper, ribbon, and colorful wrapping paper had to be purchased.

From the balance of the money, each child was given 50 cents and all trouped to the local 5–10–15 cent store to buy gifts. As readiness, the children reviewed the value of money, how change is made, and the kind of item which would be suitable for gifts for children who are hospitalized. Lists were written on the blackboard: color books, magic slates, clay, books, puzzles, paper dolls, etc., and each child chose beforehand a gift to buy, and then made a selection at the store.

Returning to class, each child showed his gift, told how much it had cost, including tax, and how much change was received from the original 50 cents. One child said, "It looks as if we now have more money in change than we had when we started."

"Let's count it and see," said another. After counting all the pennies and nickels, it was discovered that the class had but $5.51 left. The balance was used to subscribe to a children's magazine and to buy a large stuffed Easter bunny. The bunny was not to be given away, but was to be left at the hospital. A daily record of the money spent was written on the blackboard as follows:

Potato basket	$ .50
Bags for candy	2.07
Wrapping paper for presents	4.13
Crepe paper for Easter basket	.47
Bunny	2.51
Magazine	3.00
Presents	10.92
Total	$23.60

With the decorated basket filled with presents, the children, teacher and local doctor visited the hospital.

Later the hospital staff wrote a thank-you letter to the class, and the children learned the big bunny (named Candy by the children) was given to the sick children to hold in their arms on the way to the operating room. Since each child wrote his name on the present he bought and wrapped, many children have received individual thank-you letters.

The local radio station learned of this activity, and the class was invited to tell of their experience over the radio. Each child was given the opportunity to tell something about this project. Appropriate songs were sung, along with this original song about the trip to the 5–10–15 cent store.

Melody: (School Days)

FUN AT THE STORE

(Chorus)

School Days, School Days,
Dear old Northmead school days,
Reading and Writing and Arithmetic,
We've learned to make change, it's really no trick,
We went shopping a few days ago,
Down to the dime store we wanted to go,
To buy for the hospital gifts for the sick,
And hope their recovery'll be quick.

An arithmetic project can be fun for children and at the same time be filled with desirable learnings.

The Arithmetic Textbook

PRACTICALLY ALL American textbooks in arithmetic are well manufactured. They will withstand considerable handling and use. Likewise most of them have a neat and attractive appearance. Usually they are most carefully written and edited. The language is carefully scaled to the age level of pupils. The type size and topography is carefully chosen. New ideas and words are introduced according to a definite plan. The exercises are carefully graded and the sequence of topics is developed in terms of both the logical structure of the mathematics involved and the maturity levels of the children.

Many of our series of arithmetic textbooks are accompanied with teacher's manuals. Some of these manuals are better than the typical course in arithmetic methods which so often constitutes the major training of a teacher. The teacher's manual is particularly appreciated by beginning teachers and by those who are returning to the profession after having been out of the classroom for some years. A good manual will offer many ideas to any good teacher who is seeking to improve her service. Further, a good manual is a valuable aid to a teacher who does not wish to confine the learning of arithmetic to textbook procedures. Much of our best arithmetic can never be written into a textbook. In studying a new textbook series it is often the manual that reveals the real insight of arithmetic learning held by the authors.

The textbook must be considered more than a "drill book." It is in fact a planned sequence of growth in arithmetic learning. To some teachers it is a convenient outline of the work she does with the pupils but much of her work comes from without the book. To her the book is however a valuable aid. To other teachers a textbook is primarily a reference book in which pupils find an explanation of ideas, a wealth of information, and modes of working with numbers. To some other teachers, the textbook is to be studied in order to learn how to "do arithmetic."

Methods of teaching and learning are not usually suggested in textbooks. It is the teacher who must determine how best the individuals in her class can learn the kind of arithmetic she thinks it important that they do learn. However, some textbooks are better suited to discovery procedures while others are better suited to drill and memorization. A textbook should be chosen in terms of the aims, the procedures, and the educational vision of the teacher and the school for the pupils in that school. Thus in selecting a series of books teachers should make a careful study to see if (a) the books will help to teach arithmetic and (b) the books will help the pupils to understand and to learn arithmetic. This is not a simple task. It requires a good deal of study and insight to make a wise choice.

Ben's Puzzles

When Rita asked Ben for half a piece of paper he said that this was impossible for him to do because as soon as he tore a piece to give her half, it was a piece of paper he gave her and not a half piece. How can he give her a half piece?

When asked for a name for the child the parson was told to name him "Psalm Sieve" which was taken from the Bible as were the names for his brothers and sisters. The parson was puzzled and asked to see the name in the bible. The proud parents showed him in capital letters PSALM CIV.